## Chapter 3 Quadratic Functions

### Section 3.1 Investigating Quadratic Functions in Vertex Form

**Section 3.1 Page 157 Question 1**

**a)** The graph of $f(x) = 7x^2$ will open upward and be narrower than the graph of $f(x) = x^2$, since $a > 1$. The parabola will have a minimum value and a range of $\{y \mid y \geq 0, y \in \mathrm{R}\}$.

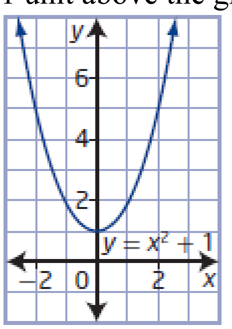
**b)** The graph of $f(x) = \frac{1}{6}x^2$ will open upward and be wider than the graph of $f(x) = x^2$, since $0 < a < 1$. The parabola will have a minimum value and a range of $\{y \mid y \geq 0, y \in \mathrm{R}\}$.

**c)** The graph of $f(x) = -4x^2$ will open downward and be narrower than the graph of $f(x) = x^2$, since $a < -1$. The parabola will have a maximum value and a range of $\{y \mid y \leq 0, y \in \mathrm{R}\}$.

**d)** The graph of $f(x) = -0.2x^2$ will open downward and be wider than the graph of $f(x) = x^2$, since $-1 < a < 0$. The parabola will have a maximum value and a range of $\{y \mid y \leq 0, y \in \mathrm{R}\}$.

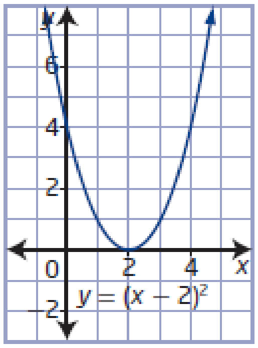
**Section 3.1 Page 157 Question 2**

**a)** $y = x^2$ and $y = x^2 + 1$
The shapes of the graphs are the same. Since $q = 1$ for $y = x^2 + 1$, its graph is translated 1 unit above the graph of $y = x^2$.



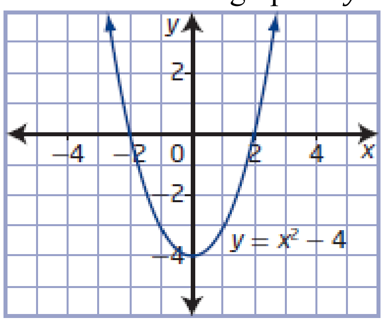

vertex: (0, 1)
axis of symmetry: $x = 0$
domain: $\{x \mid x \in \mathrm{R}\}$
range: $\{y \mid y \geq 1, y \in \mathrm{R}\}$
$x$-intercepts: none
$y$-intercept: 1

**b)** $y = x^2$ and $y = (x - 2)^2$
The shapes of the graphs are the same. Since $p = 2$ for $y = (x - 2)^2$, its graph is translated 2 units to the right of the graph of $y = x^2$.



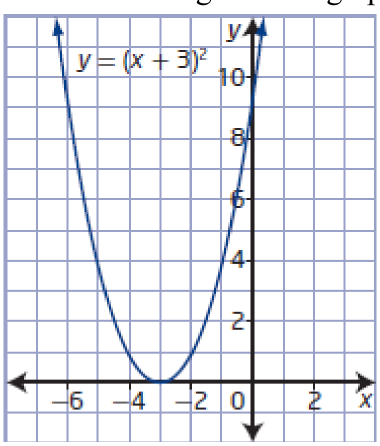

vertex: (2, 0)
axis of symmetry: $x = 2$
domain: $\{x \mid x \in \mathrm{R}\}$
range: $\{y \mid y \geq 0, y \in \mathrm{R}\}$
$x$-intercept: 2
$y$-intercept: 4

**c)** $y = x^2$ and $y = x^2 - 4$
The shapes of the graphs are the same. Since $q = -4$ for $y = x^2 - 4$, its graph is translated 4 units below the graph of $y = x^2$.




vertex: (0, –4)
axis of symmetry: $x = 0$
domain: $\{x \mid x \in \mathrm{R}\}$
range: $\{y \mid y \geq -4, y \in \mathrm{R}\}$
$x$-intercepts: –2 and 2
$y$-intercept: –4

**d)** $y = x^2$ and $y = (x + 3)^2$
The shapes of the graphs are the same. Since $p = -3$ for $y = (x + 3)^2$, its graph is translated 3 units to the right of the graph of $y = x^2$.




vertex: (–3, 0)
axis of symmetry: $x = -3$
domain: $\{x \mid x \in \mathrm{R}\}$
range: $\{y \mid y \geq 0, y \in \mathrm{R}\}$
$x$-intercept: –3
$y$-intercept: 9

**Section 3.1 Page 157 Question 3**

**a)** For $f(x) = (x + 5)^2 + 11$, $a = 1$, $p = -5$, and $q = 11$. Since $a = 1$, the shape of the graph is the same as the graph of $f(x) = x^2$. Since $p = -5$ and $q = 11$, the vertex is located at (–5, 11).

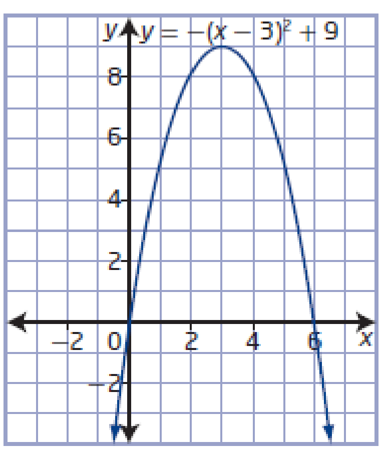
To sketch the graph of $f(x) = (x + 5)^2 + 11$, transform the graph of $f(x) = x^2$ by translating 5 units to the left and 11 units up.

**b)** For $f(x) = -3x^2 - 10$, $a = -3$, $p = 0$, and $q = -10$. Since $a < -1$, the shape of the graph is narrower than the graph of $f(x) = x^2$ and opens downward. Since $p = 0$ and $q = -10$, the vertex is located at (0, –10).
To sketch the graph of $f(x) = -3x^2 - 10$, transform the graph of $f(x) = x^2$ by
• multiplying the $y$-values by a factor of 3
• reflecting in the $x$-axis
• translating 10 units down

**c)** For $f(x) = 5(x + 20)^2 - 21$, $a = 5$, $p = -20$, and $q = -21$. Since $a > 1$, the shape of the graph is narrower than the graph of $f(x) = x^2$ and opens upward. Since $p = -20$ and $q = -21$, the vertex is located at (–20, –21).
To sketch the graph of $f(x) = 5(x + 20)^2 - 21$, transform the graph of $f(x) = x^2$ by
• multiplying the $y$-values by a factor of 5
• translating 20 units to the left and 21 units down

**d)** For $f(x) = -\frac{1}{8}(x - 5.6)^2 + 13.8$, $a = -\frac{1}{8}$, $p = 5.6$, and $q = 13.8$. Since $-1 < a < 0$, the shape of the graph is wider than the graph of $f(x) = x^2$ and opens downward. Since $p = 5.6$ and $q = 13.8$, the vertex is located at (5.6, 13.8).
To sketch the graph of $f(x) = -\frac{1}{8}(x - 5.6)^2 + 13.8$, transform the graph of $f(x) = x^2$ by
• multiplying the $y$-values by a factor of $\frac{1}{8}$
• reflecting in the $x$-axis
• translating 5.6 units to the right and 13.8 units up

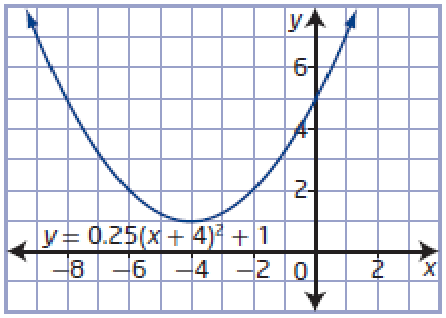
**Section 3.1 Page 157 Question 4**




**a)** For $y = -(x - 3)^2 + 9$, $a = -1$, $p = 3$, and $q = 9$.

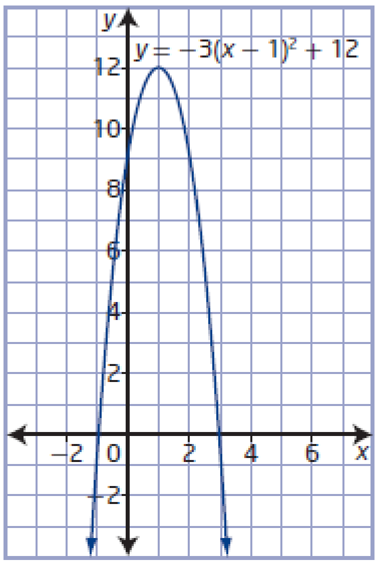
vertex: (3, 9)
axis of symmetry: $x = 3$
opens downward
maximum value of 9
domain: $\{x \mid x \in R\}$
range: $\{y \mid y \leq 9, y \in R\}$
$x$-intercepts: 0 and 6
$y$-intercept: 0

**b)** For $y = 0.25(x + 4)^2 + 1$, $a = 0.25$, $p = -4$, and $q = 1$.



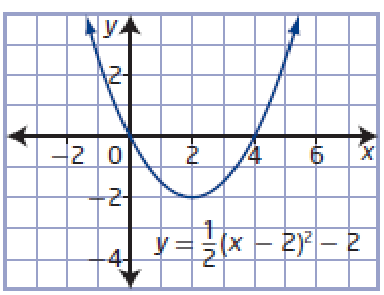

vertex: (−4, 1)
axis of symmetry: $x = -4$
opens upward
minimum value of 1
domain: $\{x \mid x \in \mathrm{R}\}$
range: $\{y \mid y \geq 1, y \in \mathrm{R}\}$
*x*-intercepts: none
*y*-intercept: 5

**c)** For $y = -3(x - 1)^2 + 12$, $a = -3$, $p = 1$, and $q = 12$.




vertex: (1, 12)
axis of symmetry: $x = 1$
opens downward
maximum value of 12
domain: $\{x \mid x \in \mathrm{R}\}$
range: $\{y \mid y \leq 12, y \in \mathrm{R}\}$
*x*-intercepts: –1 and 3
*y*-intercept: 9

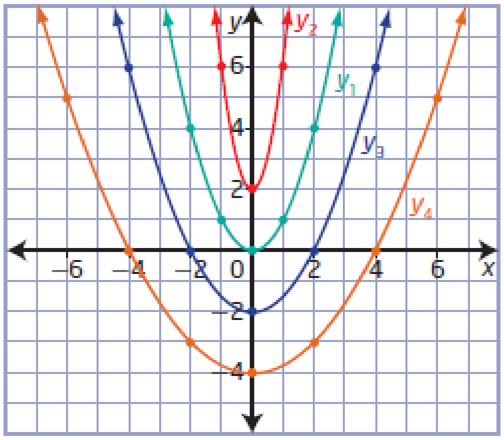
**d)** For $y = \frac{1}{2}(x - 2)^2 - 2$, $a = \frac{1}{2}$, $p = 2$, and $q = -2$.




vertex: (2, –2)
axis of symmetry: $x = 2$
opens upward
minimum value of –2
domain: $\{x \mid x \in \mathrm{R}\}$
range: $\{y \mid y \geq -2, y \in \mathrm{R}\}$
*x*-intercepts: 0 and 4
*y*-intercept: 0

**Section 3.1 Page 157 Question 5**

**a)** Use points and substitution to determine a quadratic function in vertex form, $y = a(x - p)^2 + q$, for each parabola.




• The vertex of $y_1$ is located at (0, 0), so $p = 0$ and $q = 0$. Since the parabola opens upward, $a > 0$. Then, $y_1 = a(x - 0)^2 + 0$ or $y_1 = ax^2$. Substitute the coordinates of another point on the graph to find $a$. Choose (1, 1).

$1 = a(1)^2$

$a = 1$

The quadratic function in vertex form is $y_1 = x^2$.

• The vertex of $y_2$ is located at (0, 2), so $p = 0$ and $q = 2$. Since the parabola opens upward, $a > 0$. Then, $y_2 = a(x - 0)^2 + 2$ or $y_2 = ax^2 + 2$. Use (1, 6) to find $a$.

$6 = a(1)^2 + 2$

$a = 4$

The quadratic function in vertex form is $y_2 = 4x^2 + 2$.

• The vertex of $y_3$ is located at (0, –2), so $p = 0$ and $q = –2$. Since the parabola opens upward, $a > 0$. Then, $y_3 = a(x - 0)^2 - 2$ or $y_3 = ax^2 - 2$. Use (2, 0) to find $a$.

$0 = a(2)^2 - 2$

$a = \frac{1}{2}$

The quadratic function in vertex form is $y_3 = \frac{1}{2}x^2 - 2$.

• The vertex of $y_4$ is located at (0, –4), so $p = 0$ and $q = –4$. Since the parabola opens upward, $a > 0$. Then, $y_4 = a(x - 0)^2 - 4$ or $y_4 = ax^2 - 4$. Use (2, –3) to find $a$.

$-3 = a(2)^2 - 4$

$a = \frac{1}{4}$

The quadratic function in vertex form is $y_4 = \frac{1}{4}x^2 - 4$.

**b)** For parabolas with the same shape and vertex but open downward, multiply the value of $a$ by –1.

$y_1 = -x^2$, $y_2 = -4x^2 + 2$, $y_3 = -\frac{1}{2}x^2 - 2$, $y_4 = -\frac{1}{4}x^2 - 4$

**c)** For parabolas with the same shape but translated 4 units to left, add 4 to each value of $p$.

$y_1 = (x + 4)^2$, $y_2 = 4(x + 4)^2 + 2$, $y_3 = \frac{1}{2}(x + 4)^2 - 2$, $y_4 = \frac{1}{4}(x + 4)^2 - 4$

**d)** For parabolas with the same shape but translated 2 units down, subtract 2 from each value of $q$.

$y_1 = x^2 - 2$, $y_2 = 4x^2$, $y_3 = \frac{1}{2}x^2 - 4$, $y_4 = \frac{1}{4}x^2 - 6$

**Section 3.1 Page 157 Question 6**

For $f(x) = 5(x - 15)^2 - 100$, $a = 5$, $p = 15$, and $q = -100$.
**a)** The vertex is located at $(p, q)$, or $(15, -100)$.

**b)** The equation of the axis of symmetry is $x = p$, or $x = 15$.

**c)** Since $a > 0$, the graph opens upward.

**d)** Since $a > 0$, the graph has a minimum value of $q$, or –100.

**e)** The domain is $\{x \mid x \in \mathrm{R}\}$. Since the function has a minimum value of –100, the range is $\{y \mid y \geq -100, y \in \mathrm{R}\}$.

**f)** Since the graph has a minimum value of –100, which is below the $x$-axis, and opens upward, there are two $x$-intercepts.

**Section 3.1 Page 158 Question 7**

**a)** For $y = -4x^2 + 14$, $a = -4$, $p = 0$, and $q = 14$.
The vertex is located at $(p, q)$, or $(0, 14)$.
The equation of the axis of symmetry is $x = p$, or $x = 0$.
Since $a < 0$, the graph opens downward and has a maximum value of $q$, or 14.
The domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \leq 14, y \in \mathrm{R}\}$.
Since the graph has a maximum value of 14, which is above the $x$-axis, and opens downward, there are two $x$-intercepts.

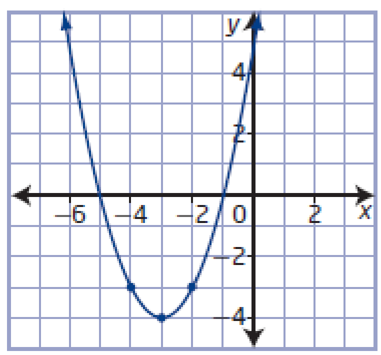
**b)** For $y = (x + 18)^2 - 8$, $a = 1$, $p = -18$, and $q = -8$.
The vertex is located at (–18, –8).
The equation of the axis of symmetry is $x = -18$.
Since $a > 0$, the graph opens upward and has a minimum value of –8.
The domain is $\{x \mid x \in R\}$ and the range is $\{y \mid y \geq -8, y \in R\}$.
Since the graph has a minimum value of –8, which is below the *x*-axis, and opens upward, there are two *x*-intercepts.

**c)** For $y = 6(x - 7)^2$, $a = 6$, $p = 7$, and $q = 0$.
The vertex is located at (7, 0).
The equation of the axis of symmetry is $x = 7$.
Since $a > 0$, the graph opens upward and has a minimum value of 0.
The domain is $\{x \mid x \in R\}$ and the range is $\{y \mid y \geq 0, y \in R\}$.
Since the graph has a minimum value of 0, which is on the *x*-axis, and opens upward, there is one *x*-intercept.

**d)** For $y = -\frac{1}{9}(x + 4)^2 - 36$, $a = -\frac{1}{9}$, $p = -4$, and $q = -36$.
The vertex is located at (–4, –36).
The equation of the axis of symmetry is $x = -4$.
Since $a < 0$, the graph opens downward and has a maximum value of –36.
The domain is $\{x \mid x \in R\}$ and the range is $\{y \mid y \leq -36, y \in R\}$.
Since the graph has a maximum value of –36, which is below the *x*-axis, and opens downward, there are no *x*-intercepts.

**Section 3.1 Page 158 Question 8**

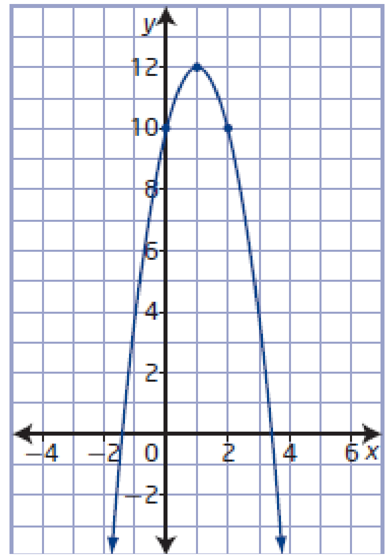
**a)** Since the vertex is located at (–3, –4), $p = -3$ and $q = -4$.
So, the function is of the form $y = a(x + 3)^2 - 4$. Substitute (–2, –3) and solve for *a*.
$-3 = a(-2 + 3)^2 - 4$
$-3 = a - 4$
$a = 1$
The quadratic function in vertex form is $y = (x + 3)^2 - 4$.



**b)** Since the vertex is located at (1, 12), $p = 1$ and $q = 12$. So, the function is of the form $y = a(x - 1)^2 + 12$. Substitute (0, 10) and solve for $a$.

$10 = a(0 - 1)^2 + 12$
$10 = a + 12$
$a = -2$

The quadratic function in vertex form is $y = -2(x - 1)^2 + 12$.

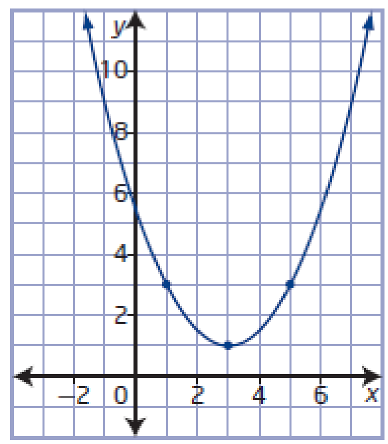



**c)** Since the vertex is located at (3, 1), $p = 3$ and $q = 1$. So, the function is of the form $y = a(x - 3)^2 + 1$. Substitute (1, 3) and solve for $a$.

$3 = a(1 - 3)^2 + 1$
$3 = 4a + 1$
$a = \frac{1}{2}$

The quadratic function in vertex form is

$y = \frac{1}{2}(x - 3)^2 + 1$.

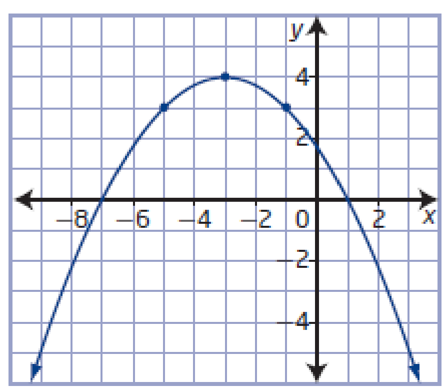



**d)** Since the vertex is located at (–3, 4), $p = -3$ and $q = 4$. So, the function is of the form $y = a(x + 3)^2 + 4$. Substitute (–1, 3) and solve for $a$.

$3 = a(-1 + 3)^2 + 4$
$3 = 4a + 4$
$a = -\frac{1}{4}$

The quadratic function in vertex form is

$y = -\frac{1}{4}(x + 3)^2 + 4$.



**Section 3.1 Page 158 Question 9**

**a)** vertex at (0, 0), passing through the point (6, −9)
Since $p = 0$ and $q = 0$, the function is of the form $y = ax^2$.
Substitute the coordinates of the given point to find $a$.

$$-9 = a(6)^2$$
$$-9 = 36a$$
$$a = -\frac{1}{4}$$

The quadratic function in vertex form with the given characteristics is $y = -\frac{1}{4}x^2$.

**b)** vertex at (0, –6), passing through the point (3, 21)
Since $p = 0$ and $q = –6$, the function is of the form $y = ax^2 – 6$.
Substitute the coordinates of the given point to find $a$.

$$21 = a(3)^2 - 6$$
$$27 = 9a$$
$$a = 3$$

The quadratic function in vertex form with the given characteristics is $y = 3x^2 – 6$.

**c)** vertex at (2, 5), passing through the point (4, –11)
Since $p = 2$ and $q = 5$, the function is of the form $y = a(x – 2)^2 + 5$.
Substitute the coordinates of the given point to find $a$.

$$-11 = a(4 - 2)^2 + 5$$
$$-16 = 4a$$
$$a = -4$$

The quadratic function in vertex form with the given characteristics is $y = –4(x – 2)^2 + 5$.

**d)** vertex at (–3, –10), passing through the point (2, –5)
Since $p = –3$ and $q = –10$, the function is of the form $y = a(x + 3)^2 – 10$.
Substitute the coordinates of the given point to find $a$.

$$-5 = a(2 + 3)^2 - 10$$
$$5 = 25a$$
$$a = \frac{1}{5}$$

The quadratic function in vertex form with the given characteristics is $y = \frac{1}{5}(x + 3)^2 – 10$.

**Section 3.1 Page 158 Question 10**

**a)** A horizontal translation of 5 units to the left causes the $x$-coordinate of the given point (4, 16) to change by –5.
$(4, 16) \rightarrow (4 - 5, 16)$ or $(-1, 16)$
Then, a vertical translation of 8 units up causes the $y$-coordinate of the point (–1, 16) to change by +8.
$(-1, 16) \rightarrow (-1, 16 + 8)$ or $(-1, 24)$

**b)** A change in width by a factor of $\frac{1}{4}$ causes the $y$-coordinate of the given point (4, 16) to change by a factor of $\frac{1}{4}$.
$(4, 16) \rightarrow \left(4, \frac{1}{4} \times 16\right)$ or $(4, 4)$
Then, a reflection in the $x$-axis causes the $y$-coordinate of the point (4, 4) to change by a factor of –1.
$(4, 4) \rightarrow (4, -1 \times 4)$ or $(4, -4)$

**c)** A reflection in the $x$-axis causes the $y$-coordinate of the given point (4, 16) to change by a factor of –1.
$(4, 16) \rightarrow (4, -1 \times 16)$ or $(4, -16)$
Then, a horizontal translation of 10 units to the right causes the $x$-coordinate of the point (4, –16) to change by +10.
$(4, -16) \rightarrow (4 + 10, -16)$ or $(14, -16)$

**d)** A change in width by a factor of 3 causes the $y$-coordinate of the given point (4, 16) to change by a factor of 3.
$(4, 16) \rightarrow (4, 3 \times 16)$ or $(4, 48)$
Then, a vertical translation of 8 units down causes the $y$-coordinate of the point (4, 48) to change by –8.
$(4, 48) \rightarrow (4, 48 - 8)$ or $(4, 40)$

**Section 3.1 Page 158 Question 11**

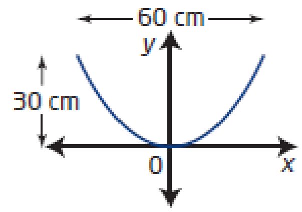
First rewrite the quadratic function $y = 20 - 5x^2$ in vertex form: $y = -5x^2 + 20$.
So, $a = -5$, $p = 0$, and $q = 20$.
To obtain the graph of $y = -5x^2 + 20$, transform the graph of $y = x^2$ by
- multiplying the $y$-values by a factor of 5
- reflecting in the $x$-axis
- translating 20 units up

**Section 3.1 Page 158 Question 12**

No. Quadratic functions will always have one *y*-intercept. Since the graphs have a domain of $\{x \mid x \in R\}$, the parabola will always intersect the *y*-axis.

**Section 3.1 Page 159 Question 13**

**a)** Let *x* and *y* represent the horizontal and vertical distances from the low point at the centre of the mirror, respectively. Since the vertex is at (0, 0), the quadratic function is of the form $y = ax^2$. From the diagram, another point on the parabola is (30, 30).




Use the coordinates of this point to find *a*.

$30 = a(30)^2$

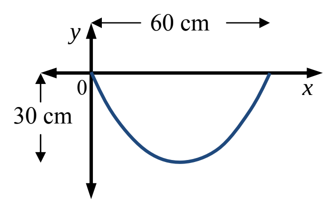
$30 = 900a$

$a = \frac{1}{30}$

A quadratic function that represents the cross-sectional shape is $y = \frac{1}{30}x^2$.

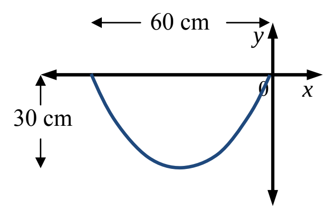
**b)** If the left outer edge of the mirror is considered the origin, then the vertex is at (30, –30) and the quadratic function becomes

$y = \frac{1}{30}(x - 30)^2 - 30$.




If the right outer edge of the mirror is considered the origin, then the vertex is at (–30, –30) and the quadratic function becomes

$y = \frac{1}{30}(x + 30)^2 - 30$.




**Section 3.1 Page 159 Question 14**

**a)** For $N(x) = -2.5(x - 36)^2 + 20\ 000$, $a = -2.5$, $p = 36$, and $q = 20\ 000$. To sketch the graph of $N(x) = -2.5(x - 36)^2 + 20\ 000$, transform the graph of $y = x^2$ by

- multiplying the *y*-values by a factor of 2.5
- reflecting in the *x*-axis
- translating 36 units to the right and 20 000 units up

The graph of $N(x) = -2.5(x - 36)^2 + 20\ 000$ has its vertex located at (36, 20 0000).
The equation of the axis of symmetry is $x = 36$.
Since $a < 0$, the graph opens downward and has a maximum value of 20 000.

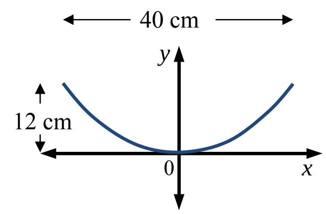
**b)** The value of $x$ that corresponds to the maximum value of the graph is 36. The optimum number of times that the commercial should be aired is 36 times.

**c)** The maximum value of the graph is the $y$-coordinate of the vertex, or 20 000. The maximum number of people that will buy this product is 20 000 people.

**Section 3.1 Page 159 Question 15**

Answers may vary. Example:

Choose the location of the origin to be the lowest point in the centre of the container. Let $x$ and $y$ represent the horizontal and vertical
distances from the low point of the container, respectively. Then, the vertex is at (0, 0) and the quadratic function is of the form $y = ax^2$. From the diagram, another point on the parabola is (20, 12).




Use the coordinates of this point to find $a$.

$12 = a(20)^2$

$12 = 400a$

$a = \dfrac{3}{100}$

A quadratic function that represents the cross-sectional shape is $y = \dfrac{3}{100}x^2$.

**Section 3.1 Page 160 Question 16**

Answers may vary. Examples:

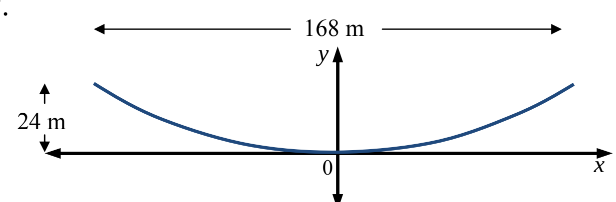
**a)** Choose the location of the origin to be the lowest point in the centre of the cables. Let $x$ and $y$ represent the horizontal and vertical distances from the low point of the cables, respectively. Then, the vertex is at (0, 0) and the quadratic function is of the form $y = ax^2$. From the diagram, another point on the parabola is (84, 24).

Use the coordinates of this point to find $a$.

$24 = a(84)^2$

$24 = 7056a$

$a = \dfrac{1}{294}$




A quadratic function that represents the shape of the cables is $y = \dfrac{1}{294}x^2$.

**b)** If the top of the left tower is considered the origin, then the vertex is at (84, –24).

Let $x$ and $y$ represent the horizontal and vertical distances from the top of the left tower, respectively. Then, the quadratic function becomes $y = \frac{1}{294}(x - 84)^2 - 24$.
If the top of the right tower is considered the origin, then the vertex is at (–84, –24).
Let $x$ and $y$ represent the horizontal and vertical distances from the top of the right tower, respectively. Then, the quadratic function becomes $y = \frac{1}{294}(x + 84)^2 - 24$.

**c)** First, determine the value of $x$ that corresponds to 35 m horizontally from one of the towers. Then, substitute and solve $y$. Finally, determine the vertical distance from the minimum value.

| $\boldsymbol{y = \frac{1}{294}x^2}$ | $\boldsymbol{y = \frac{1}{294}(x - 84)^2 - 24}$ | $\boldsymbol{y = \frac{1}{294}(x + 84)^2 - 24}$ |
|---|---|---|
| Use $x = 84 - 35$ or 49.<br>$y = \frac{1}{294}(49)^2$<br>$y = 8.166\ldots$<br>Since the minimum is 0, the vertical height of this point above the minimum is about 8.17 – 0, or 8.17 m. | Use $x = 35$.<br>$y = \frac{1}{294}(35 - 84)^2 - 24$<br>$y = -15.833\ldots$<br>Since the minimum is –24, the vertical height of this point above the minimum is about<br>–15.83 – (–24), or 8.17 m. | Use $x = -35$.<br>$y = \frac{1}{294}(-35 + 84)^2 - 24$<br>$y = -15.833\ldots$<br>Since the minimum is –24, the vertical height of this point above the minimum is about<br>–15.83 – (–24), or 8.17 m. |

The height above the minimum is the same for all three models.

**Section 3.1 Page 160 Question 17**

Determine the coordinates of the vertex for this model that has the origin at the spot above the ground from which the tennis ball was hit.
maximum height: 10 – 1 = 9
horizontal distance: 22 ÷ 2 = 11
So, the vertex is at (11, 9).
Let $x$ and $y$ represent the horizontal and vertical distances from the location that the tennis ball was hit, respectively.
Then, $p = 11$, $q = 9$, and quadratic function is of the form $y = a(x - 11)^2 + 9$.
Use (0, 0) to find $a$.

$$0 = a(0 - 11)^2 + 9$$
$$a = -\frac{9}{121}$$

A quadratic function that can be used to represent the trajectory of the tennis ball is

$$y = -\frac{9}{121}(x - 11)^2 + 9.$$

**Section 3.1 Page 160 Question 18**

Determine the coordinates of the vertex for this model that has the origin at the nozzle.
maximum height: 100 – 10 = 90
horizontal distance: 120 ÷ 2 = 60
So, the vertex is at (60, 90).
Let $x$ and $y$ represent the horizontal and vertical distances from the nozzle, respectively.
Then,
Then, $p = 60$, $q = 90$, and quadratic function is of the form
$y = a(x - 60)^2 + 90$.
Use (0, 0) to find $a$.

$0 = a(0 - 60)^2 + 90$

$-90 = 3600a$

$a = -\frac{1}{40}$

A quadratic function that can be used to represent the path of the water is

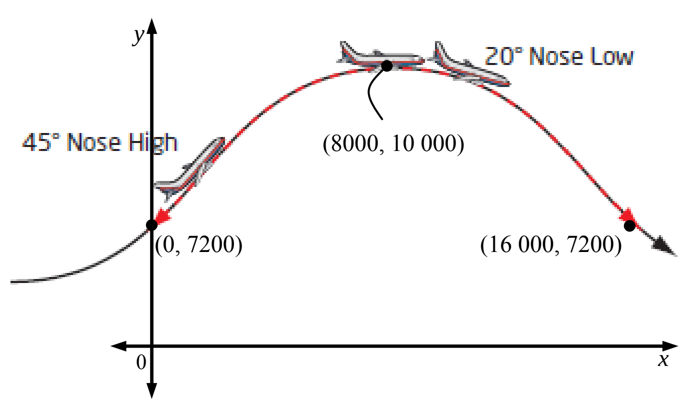
$y = -\frac{1}{40}(x - 60)^2 + 90$.

**Section 3.1 Page 161 Question 19**

Answers may vary. Example:
The function $y = x^2 + 4$ is of the form $y = x^2 + q$. Since the $q$-value is added or subtracted after squaring the $x$-value, the transformation applies directly to the parabola $y = x^2$.
So, addition represents the upward or positive direction and subtraction represents the downward or negative direction.
The function $y = (x + 4)^2$ is of the form $y = (x - p)^2$. Since the $p$-value is added or subtracted before squaring, the direction of the translation is opposite to the sign in the bracket to get back to the original $y$-value for the graph of $y = x^2$. In this case, $y = (x + 4)^2$ represents $y = (x - (-4))^2$, which is translation to the left or negative direction.

**Section 3.1 Page 161 Question 20**

**a)** Use the information given to determine the coordinates of the vertex and one other point.
Let $x$ represent the horizontal distance from the start of the parabolic path.
Let $y$ represent the vertical height above the ground.



The function is of the form
$y = a(x - 8000)^2 + 10\ 000$.
Use (0, 7200) to determine $a$.

$$7200 = a(0 - 8000)^2 + 10\ 000$$
$$-2800 = 64\ 000\ 000a$$
$$a = -\frac{7}{160\ 000}$$

A function that represents the parabolic path of the plane is

$$y = -\frac{7}{160\ 000}(x - 8000)^2 + 10\ 000.$$

**b)** For the plane's parabolic path, the domain is $\{x \mid 0 \le x \le 16\ 000, x \in \mathrm{R}\}$ and the range is $\{y \mid 7200 \le y \le 10\ 000, y \in \mathrm{R}\}$.

**Section 3.1 Page 161 Question 21**

**a)** Since the given vertex is (6, 30), $p = 6$ and $q = 30$. Then, the quadratic function is of the form $y = a(x - 6)^2 + 30$. Use the given location of the $y$-intercept at (0, –24) to find $a$.

$$-24 = a(0 - 6)^2 + 30$$
$$-54 = 36a$$
$$a = -\frac{3}{2}$$

A quadratic function with the given characteristics is $y = -\frac{3}{2}(x - 6)^2 + 30$.

**b)** Since the given minimum value is –24, $q = -24$. Due to symmetry, the $x$-value midway between given $x$-intercepts, –21 and –5, is the $x$-coordinate of the vertex.

$$\frac{-21 + (-5)}{2} = -13$$

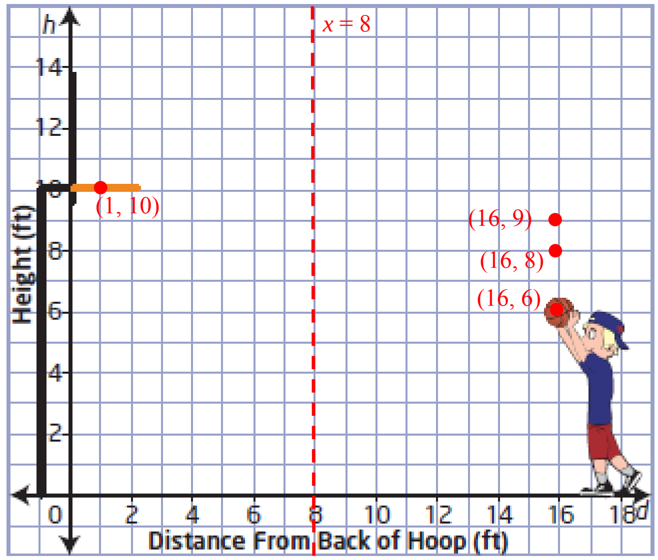
Then, $p = -13$ and the quadratic function is of the form $y = a(x + 13)^2 - 24$. Use (–5, 0) to find $a$.

$$0 = a(-5 + 13)^2 - 24$$
$$24 = 64a$$
$$a = \frac{3}{8}$$

A quadratic function with the given characteristics is $y = \frac{3}{8}(x + 13)^2 - 24$.

## Section 3.1 Page 161 Question 22

Answers may vary. Examples:
Chose the axis of symmetry to be $x = 8$, the location of the hoop to be (1, 10), and three release heights for a distance of 16 ft from the hoop.




For each scenario, substitute the coordinates of the hoop into the function $y = a(x - 8)^2 + q$ to get a linear equation involving the variables $a$ and $q$. Repeat with the coordinates of the release point. Then, solve the system of linear equations.

For (16, 6), the system of equations is

$$10 = 49a + q \quad ①$$
$$\underline{6 = 64a + q} \quad ②$$
$$4 = -15a \quad ① - ②$$
$$a = -\frac{4}{15}$$

Substitute the $a$-value into equation ① to find $q$.

$$10 = 49\left(-\frac{4}{15}\right) + q$$
$$10 = -\frac{196}{15} + q$$
$$q = \frac{346}{15}$$

A quadratic function that represents a possible trajectory of the basketball is

$$y = -\frac{4}{15}(x - 8)^2 + \frac{346}{15}.$$

For (16, 8), the system of equations is

$$10 = 49a + q \quad ①$$
$$\underline{8 = 64a + q} \quad ②$$
$$2 = -15a \quad ① - ②$$
$$a = -\frac{2}{15}$$

Substitute the $a$-value into equation ① to find $q$.

$$10 = 49\left(-\frac{2}{15}\right) + q$$

$q = \frac{248}{15}$

A quadratic function that represents a possible trajectory of the basketball is $y = -\frac{2}{15}(x - 8)^2 + \frac{248}{15}$.

For (16, 9), the system of equations is

$$\begin{aligned} 10 &= 49a + q \quad ① \\ 9 &= 64a + q \quad ② \\ \hline 1 &= -15a \quad ① - ② \\ a &= -\frac{1}{15} \end{aligned}$$

Substitute the $a$-value into equation ① to find $q$.

$10 = 49\left(-\frac{1}{15}\right) + q$

$q = \frac{199}{15}$

A quadratic function that represents a possible trajectory of the basketball is $y = -\frac{1}{15}(x - 8)^2 + \frac{199}{15}$.

**b)** The function $y = -\frac{2}{15}(x - 8)^2 + \frac{248}{15}$ is the most realistic trajectory, since it allows for the person to jump before releasing the basketball.

**c)** For the function chosen in part b), a realistic domain and range are $\{x \mid 0 \le x \le 16, x \in \mathrm{R}\}$ and $\left\{y \mid 0 \le y \le \frac{248}{15}, y \in \mathrm{R}\right\}$, respectively.

**Section 3.1 Page 161 Question 23**

The function $f(x) = a(x - p)^2 + q$ represents a change in width of factor of $a$ and a translation of $p$ units horizontally and $q$ units vertically.

A change in width by a factor of $a$ causes the $y$-coordinate of the given point $(m, n)$ to change by a factor of $a$.

$(m, n) \to (m, an)$

Next, a horizontal translation of $p$ units causes the $x$-coordinate of the point $(m, an)$ to change by $p$.

$(m, an) \to (m + p, an)$

Then, a vertical translation of $q$ units causes the $y$-coordinate of the point $(m + p, an)$ to change by $q$.

$(m + p, an) \to (m + p, an + q)$

**Section 3.1 Page 162 Question 24**

Answers may vary. Examples:

**a)** $f(x) = -3(x - 2)^2 + 4$

**b)** Sketch the graph using points and symmetry. Plot the vertex (2, 4) and draw the line of symmetry, $x = 2$. Determine another point on the curve, say the $y$-intercept, which occurs at
(0, –8). Next, find its corresponding point, which is (4, –8). Plot the two additional points and complete the sketch of the parabola.

**Section 3.1 Page 162 Question 25**

Answers may vary. Example:

Visualize the location of the vertex from the values of $p$ and $q$ and direction of the opening from the value of $a$.

| Location of Vertex | Direction of Opening | Number of $x$-Intercepts | Example |
|---|---|---|---|
| Below the $x$-axis | upward | 2 | $f(x) = 2(x - 1)^2 - 5$ |
| | downward | 0 | $f(x) = -2(x - 1)^2 - 5$ |
| On the $x$-axis | upward | 1 | $f(x) = (x - 3)^2$ |
| | downward | 1 | $f(x) = -(x + 3)^2$ |
| Above the $x$-axis | upward | 0 | $f(x) = 3(x + 2)^2 + 4$ |
| | downward | 2 | $f(x) = -3(x + 2)^2 + 4$ |

**Section 3.1 Page 162 Question 26**

Answers may vary. Example:

**Steps 1 to 3:**

I used these restricted linear and quadratic functions:

$x = 2$, $\{y \mid -5 \le y \le 5, y \in \mathrm{R}\}$ in pink

$x = 6$, $\{y \mid -5 \le y \le 5, y \in \mathrm{R}\}$ in pink

$y = 5$, $\{x \mid 2 \le x \le 6, x \in \mathrm{R}\}$ in blue

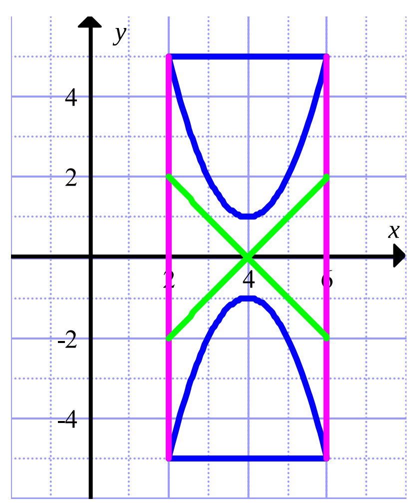
$y = -5$, $\{x \mid 2 \le x \le 6, x \in \mathrm{R}\}$ in blue

$y = -x + 4$, $\{x \mid 2 \le x \le 6, x \in \mathrm{R}\}$ in green

$y = x - 4$, $\{x \mid 2 \le x \le 6, x \in \mathrm{R}\}$ in green

$y = (x - 4)^2 + 1$, $\{x \mid 2 \le x \le 6, x \in \mathrm{R}\}$ in blue

$y = -(x - 4)^2 - 1$, $\{x \mid 2 \le x \le 6, x \in \mathrm{R}\}$ in blue



## Section 3.2 Investigating Quadratic Functions in Standard Form

**Section 3.2 Page 174 Question 1**

**a)** The function $f(x) = 2x^2 + 3x$ is quadratic, since it is a polynomial of degree two.

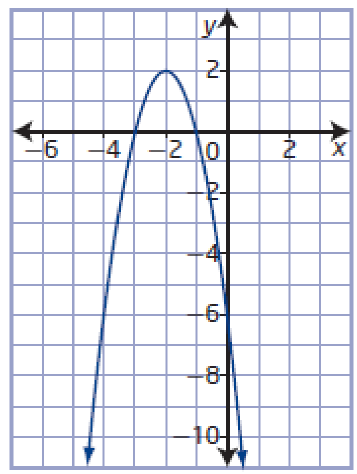
**b)** The function $f(x) = 5 – 3x$ is not quadratic, since it is a polynomial of degree one.

**c)** The function $f(x) = x(x + 2)(4x – 1)$ is not quadratic, since when expanded it is a polynomial of degree three.

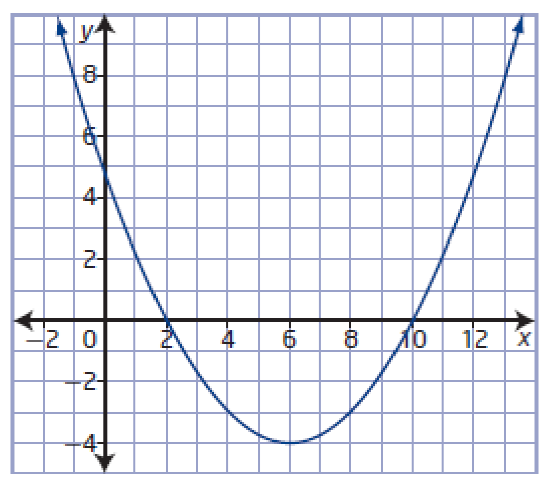
**d)** The function $f(x) = (2x – 5)(3x – 2)$ is quadratic, since when expanded it is a polynomial of degree two.

**Section 3.2 Page 174 Question 2**

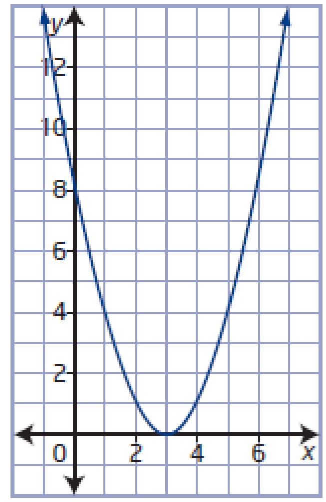
**a)** The coordinates of the vertex are (–2, 2).
The equation of the axis of symmetry is $x = –2$.
The $x$-intercepts are –3 and –1, and the $y$-intercept is –6.
The graph has a maximum value of 2, since the parabola opens downward.
The domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \le 2, y \in \mathrm{R}\}$.




**b)** The coordinates of the vertex are (6, –4).
The equation of the axis of symmetry is $x = 6$.
The $x$-intercepts are 2 and 10, and the $y$-intercept is 5.
The graph has a minimum value of –4, since the parabola opens upward.
The domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \ge –4, y \in \mathrm{R}\}$.



**c)** The coordinates of the vertex are (3, 0).
The equation of the axis of symmetry is $x = 3$.
The $x$-intercept is 3, and the $y$-intercept is 8.
The graph has a minimum value of 0, since the parabola opens upward.
The domain is $\{x \mid x \in R\}$ and the range is $\{y \mid y \geq 0, y \in R\}$.




**Section 3.2 Page 174 Question 3**

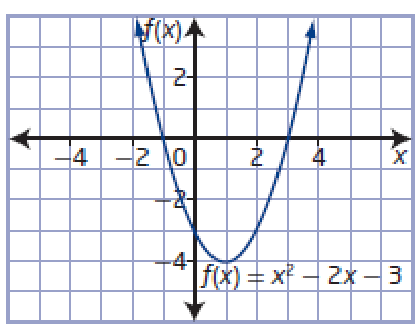
**a)** Expand $f(x) = 5x(10 – 2x)$ and write in standard form.
$f(x) = 5x(10 – 2x)$
$f(x) = 50x – 10x^2$
$f(x) = –10x^2 + 50x$

**b)** Expand $f(x) = (10 – 3x)(4 – 5x)$ and write in standard form.
$f(x) = (10 – 3x)(4 – 5x)$
$f(x) = 40 – 50x – 12x + 15x^2$
$f(x) = 15x^2 – 62x + 40$

**Section 3.2 Page 174 Question 4**

**a)**

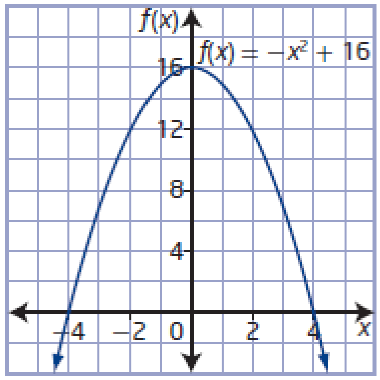
| $x$ | $f(x) = x^2 – 2x – 3$ |
|---|---|
| –1 | $f(–1) = (–1)^2 – 2(–1) – 3$<br>$= 0$ |
| 0 | $f(0) = 0^2 – 2(0) – 3$<br>$= –3$ |
| 1 | $f(1) = 1^2 – 2(1) – 3$<br>$= –4$ |
| 2 | $f(2) = 2^2 – 2(2) – 3$<br>$= –3$ |
| 3 | $f(3) = 3^2 – 2(3) – 3$<br>$= 0$ |




vertex: (1, –4)
axis of symmetry: $x = 1$
opens upward
minimum value: –4
domain: $\{x \mid x \in R\}$
range: $\{y \mid y \geq –4, y \in R\}$
$x$-intercepts: –1 and 3
$y$-intercept: –3

**b)**

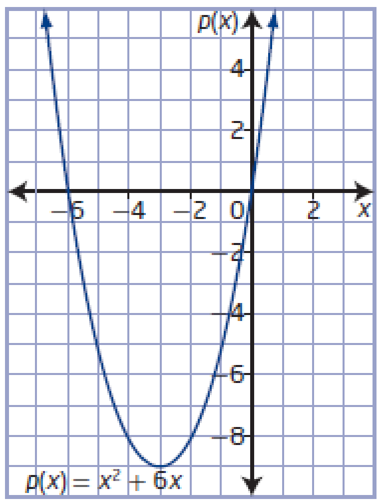
| $x$ | $f(x) = -x^2 + 16$ |
|---|---|
| –4 | $f(-4) = -(-4)^2 + 16$<br>$= 0$ |
| –2 | $f(-2) = -(-2)^2 + 16$<br>$= 12$ |
| 0 | $f(0) = -(0)^2 + 16$<br>$= 16$ |
| 2 | $f(2) = -(2)^2 + 16$<br>$= 12$ |
| 4 | $f(4) = -(4)^2 + 16$<br>$= 0$ |




vertex: (0, 16)
axis of symmetry: $x = 0$
opens downward
maximum value: 16
domain: $\{x \mid x \in \mathrm{R}\}$
range:
$\{y \mid y \leq 16, y \in \mathrm{R}\}$
$x$-intercepts: –4 and 4
$y$-intercept: 16

**c)**

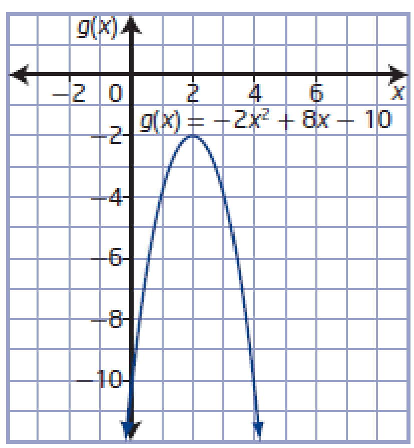
| $x$ | $f(x) = x^2 + 6x$ |
|---|---|
| –6 | $f(-6) = (-6)^2 + 6(-6)$<br>$= 0$ |
| –4 | $f(-4) = (-4)^2 + 6(-4)$<br>$= -8$ |
| –3 | $f(-3) = (-3)^2 + 6(-3)$<br>$= -9$ |
| –2 | $f(-2) = (-2)^2 + 6(-2)$<br>$= -8$ |
| 0 | $f(0) = 0^2 + 6(0)$<br>$= 0$ |




vertex: (–3, –9)
axis of symmetry: $x = -3$
opens upward
minimum value: –9
domain: $\{x \mid x \in \mathrm{R}\}$
range:
$\{y \mid y \geq -9, y \in \mathrm{R}\}$;
$x$-intercepts: –6 and 0
$y$-intercept: 0

**d)**

| $x$ | $f(x) = -2x^2 + 8x - 10$ |
|---|---|
| 0 | $f(0) = -2(0)^2 + 8(0) - 10$<br>$= -10$ |
| 1 | $f(1) = -2(1)^2 + 8(1) - 10$<br>$= -4$ |
| 2 | $f(2) = -2(2)^2 + 8(2) - 10$<br>$= -2$ |
| 3 | $f(3) = -2(3)^2 + 8(3) - 10$<br>$= -4$ |
| 4 | $f(4) = -2(4)^2 + 8(4) - 10$<br>$= -10$ |

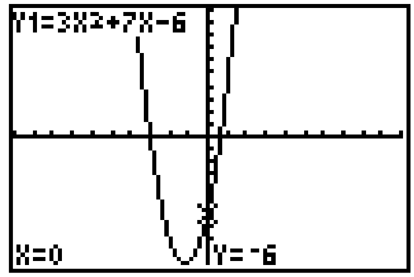



vertex: (2, –2)
axis of symmetry: $x = 2$
opens downward
maximum value: –2
domain: $\{x \mid x \in \mathrm{R}\}$
range:
$\{y \mid y \leq -2, y \in \mathrm{R}\}$
$x$-intercepts: none
$y$-intercept: –10

**Section 3.2 Page 174 Question 5**

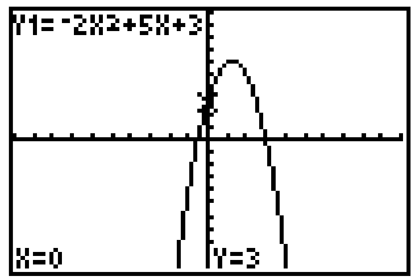
Use a graphing calculator.

**a)** Graph the function $y = 3x^2 + 7x - 6$ using window settings of $x$: [–10, 10, 1] and $y$: [–10, 10, 1].



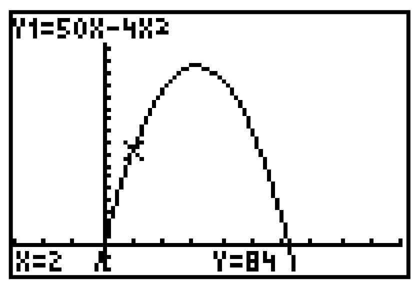

Use the minimum feature to find the vertex is located at approximately (–1.2, –10.1). So, the equation of the axis of symmetry is $x = -1.2$, and the graph opens upward with a minimum value of –10.1. The domain is $\{x \mid x \in R\}$ and the range is $\{y \mid y \geq -10.1, y \in R\}$. Use the zero feature to find the $x$-intercepts are –3 and approximately 0.7. The $y$-intercept is –6.

**b)** Graph the function $y = -2x^2 + 5x + 3$ using window settings of $x$: [–10, 10, 1] and $y$: [–10, 10, 1].



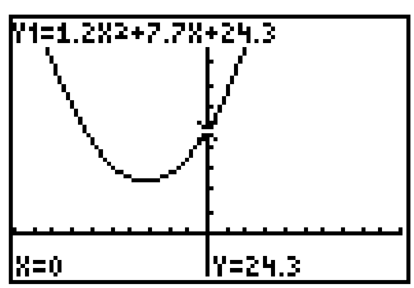

Use the maximum feature to find the vertex is located at approximately (1.3, 6.1). So, the equation of the axis of symmetry is $x = 1.3$, and the graph opens downward with a maximum value of 6.1. The domain is $\{x \mid x \in R\}$ and the range is $\{y \mid y \leq 6.1, y \in R\}$. Use the zero feature to find the $x$-intercepts are –0.5 and 3. The $y$-intercept is 3.

**c)** Graph the function $y = 50x - 4x^2$ using window settings of $x$: [–6, 20, 2] and $y$: [–20, 200, 10].




Use the maximum feature to find the vertex is located at approximately (6.3, 156.3). So, the equation of the axis of symmetry is $x = 6.3$, and the graph opens downward with a maximum value of 156.3. The domain is $\{x \mid x \in R\}$ and the range is $\{y \mid y \leq 156.3, y \in R\}$. Use the zero feature to find the $x$-intercepts are 0 and 12.5. The $y$-intercept is 0.

**d)** Graph the function $y = 1.2x^2 + 7.7x + 24.3$ using window settings of $x$: [–10, 10, 1] and $y$: [–10, 50, 5].




Use the minimum feature to find the vertex is located at approximately (–3.2, 11.9). So, the equation of the axis of symmetry is $x = -3.2$, and the graph opens upward with a minimum value of 11.9. The domain is $\{x \mid x \in R\}$ and the range is $\{y \mid y \geq 11.9, y \in R\}$. There are no $x$-intercepts and the $y$-intercept is 24.3.

**Section 3.2 Page 175 Question 6**

**a)** For $y = x^2 + 6x + 2$, $a = 1$, $b = 6$, and $c = 2$.

Use $x = \frac{-b}{2a}$ to find the $x$-coordinate of the vertex.

$x = \frac{-6}{2(1)}$

$x = -3$

Substitute $x = -3$ into $y = x^2 + 6x + 2$ to find the $y$-coordinate of the vertex.

$y = (-3)^2 + 6(-3) + 2$

$y = -7$

The vertex is located at (–3, –7).

**b)** For $y = 3x^2 - 12x + 5$, $a = 3$, $b = -12$, and $c = 5$.

Use $x = \frac{-b}{2a}$ to find the $x$-coordinate of the vertex.

$x = \frac{-(-12)}{2(3)}$

$x = 2$

Substitute $x = 2$ into $y = 3x^2 - 12x + 5$ to find the $y$-coordinate of the vertex.

$y = 3(2)^2 - 12(2) + 5$

$y = -7$

The vertex is located at (2, –7).

**c)** For $y = -x^2 + 8x - 11$, $a = -1$, $b = 8$, and $c = -11$.

Use $x = \frac{-b}{2a}$ to find the $x$-coordinate of the vertex.

$x = \frac{-8}{2(-1)}$

$x = 4$

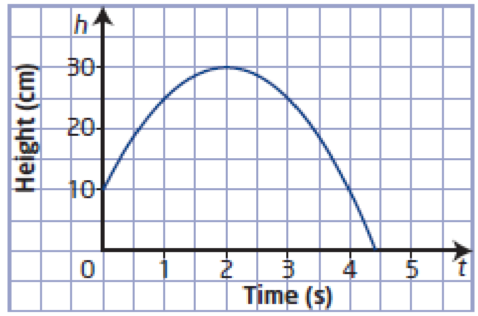
Substitute $x = 4$ into $y = -x^2 + 8x - 11$ to find the $y$-coordinate of the vertex.

$y = -(4)^2 + 8(4) - 11$

$y = 5$

The vertex is located at (4, 5).

**Section 3.2 Page 175 Question 7**




**a)** The $y$-intercept of the graph represents the height of the rock that the siksik jumped from, 10 cm.

**b)** The vertex of the graph gives the maximum height of the siksik as 30 cm at a time of 2 s.

**c)** The $x$-intercept of the graph gives the time that the siksik was in the air, or approximately 4.4 s.

**d)** The domain is $\{t \mid 0 \le t \le 4.4, t \in \mathrm{R}\}$. The range is $\{h \mid 0 \le h \le 30, h \in \mathrm{R}\}$.

**e)** Answers may vary. Example: Unlikely: the siksik rarely stay in the air for more than 4 s.

**Section 3.2 Page 175 Question 8**

**a)** For a quadratic function with an axis of symmetry of $x = 0$ and a maximum value of 8, the parabola opens downward and the vertex is (0, 8). A parabola that opens downward with a vertex above the $x$-axis has two $x$-intercepts. Since the axis of symmetry is $x = 0$, one $x$-intercept will be negative and one positive.

**b)** For a quadratic function with a vertex at (3, 1), passing through the point (1, –3), the parabola opens downward. A parabola that opens downward with a vertex above the $x$-axis has two $x$-intercepts. Since the axis of symmetry is $x = 3$ and the $x$-intercept to the left of it is positive, then the $x$-intercept to the right will also be positive.

**c)** For a quadratic function with a range of $y \ge 1$, the parabola opens upward and its vertex is above the $x$-axis. So, there are no $x$-intercepts.

**d)** For a quadratic function with a $y$-intercept of 0 and an axis of symmetry of $x = -1$, the parabola could open upward with a vertex below the $x$-axis or open downward with a vertex above the $x$-axis. For either case, there are two $x$-intercepts. One $x$-intercept, to the right of the axis of symmetry ($x = -1$), is given as zero. So, the other $x$-intercept will be to the left, or less than –1, which is negative.

**Section 3.2 Page 175 Question 9**

**a)** The domain for $f(x) = -16x^2 + 64x + 4$ is $\{x \mid x \in \mathrm{R}\}$.
To determine the range, first find the coordinates of the vertex. Substitute $a = -16$ and $b = 64$ into $x = \frac{-b}{2a}$ to find the $x$-coordinate of the vertex.

$$x = \frac{-64}{2(-16)}$$

$x = 2$

Substitute $x = 2$ into $f(x) = -16x^2 + 64x + 4$ to find the $y$-coordinate of the vertex.

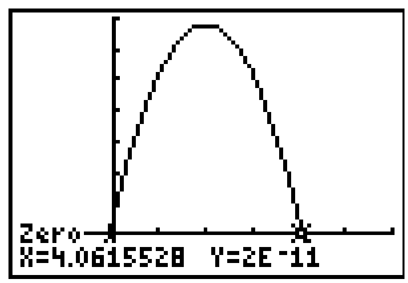
$f(2) = -16(2)^2 + 64(2) + 4$

$f(2) = 68$

The vertex is located at (2, 68).

Since $a < 0$, the parabola opens downward ans has a maximum. So, the range is $\{y \mid y \le 68, y \in \mathrm{R}\}$.

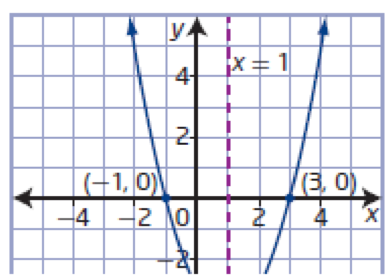
**b)** If this function represents the height of a football as a function of time, then neither height nor time can be negative. Graph $f(x) = -16x^2 + 64x + 4$ using a graphing calculator with window settings of $x$: [–2, 6, 1] and $y$: [–10, 70, 10]. Use the zero feature to determine the positive $x$-intercept is approximately 4.06. So, the domain is $\{x \mid 0 \le x \le 4.06, x \in \mathrm{R}\}$. The range is $\{y \mid 0 \le y \le 68, y \in \mathrm{R}\}$.



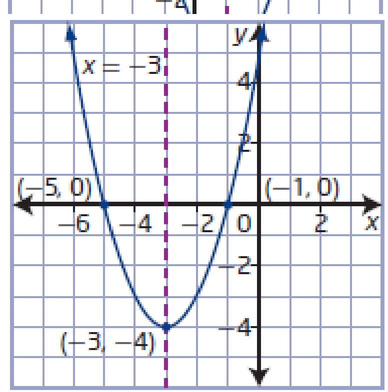

**c)** The domains and ranges are different in parts a) and b), because one represents the general case and the other represents a real-life scenario with constraints on the variables.

**Section 3.2 Page 175 Question 10**

**a)** Given $x$-intercepts at –1 and 3 and a range of $y \ge -4$, you know three points on the parabola. Two of the points are the
$x$-intercepts at (–1, 0) and (3, 0). From the two $x$-intercepts and symmetry, the $x$-coordinate of the vertex is 1. From the range, the $y$-coordinate of the vertex is –4. Then, the third point on the parabola is (1, –4).

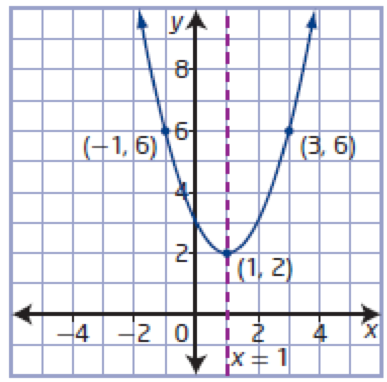



**b)** Given one $x$-intercept at –5 and vertex at (–3, –4), you know three points on the parabola. Two of the points are

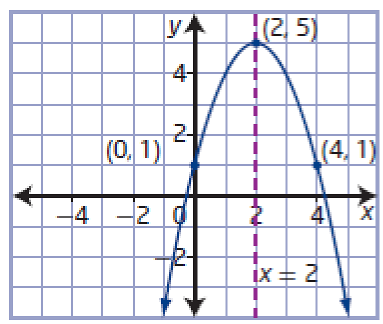


(–5, 0) and (–3, –4). Using symmetry, the third point is the other $x$-intercept at (–1, 0).

**c)** Given the axis of symmetry is $x = 1$, the minimum value of 2, and passing through (–1, 6), you know three points on the parabola. One point is given, (–1, 6). A second point is the vertex at (1, 2). Using symmetry, a third point on the parabola is (3, 6).



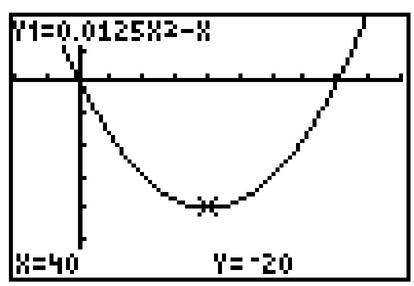

**d)** Given the vertex at (2, 5) and $y$-intercept of 1, you know three points on the parabola. Two of the points are (2, 5) and (0, 1). Using symmetry, the third point is (4, 1).




**Section 3.2 Page 176 Question 11**

**a)** Since the dish antenna is 80 cm across, the domain of $d(x) = 0.0125x^2 – x$ is $\{x \mid 0 \le x \le 80, x \in \mathrm{R}\}$.

**b)** Graph $d(x) = 0.0125x^2 – x$ using a graphing calculator with window settings of $x$: [–20, 100, 10] and $y$: [–30, 10, 5].



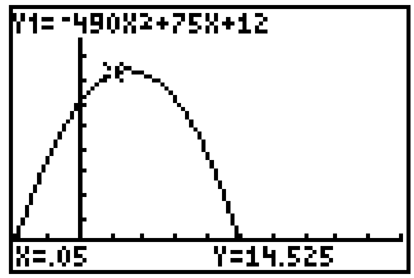

**c)** Use the minimum feature to determine the coordinates of the vertex are (40, –20). So, the maximum depth of the dish is 20 cm. This corresponds to the minimum value of the function, since the parabola opens upward.

**d)** The range of the function is $\{d \mid –20 \le d \le 0, d \in \mathrm{R}\}$.

**e)** To determine the depth of the dish at a point 25 cm from the edge, substitute $x = 25$ into
$d(x) = 0.0125x^2 - x$.
$d(25) = 0.0125(25)^2 - 25$
$d(25) = -17.1875$
The depth of the dish at a point 25 cm from the edge is approximately 17.19 cm.

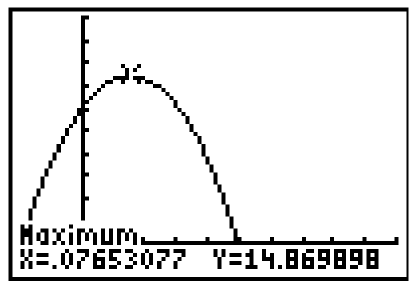
**Section 3.2 Page 176 Question 12**

**a)** Graph $h(t) = -490t^2 + 75t + 12$ using a graphing calculator with window settings of $x$: [–0.1, 0.5, 0.05] and $y$: [–2, 20, 2].

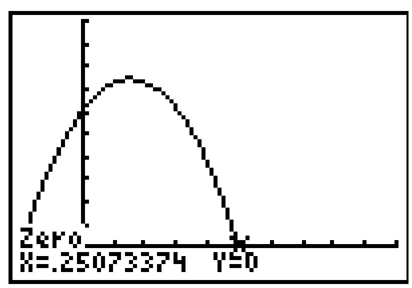



**b)** The $h$-intercept represents the height from which the spider jumped.

**c)** Use the maximum feature of the graphing calculator to find the coordinates of the vertex are approximately (0.1, 14.9). So, the spider reaches its maximum of 14.9 cm at 0.1 s.




**d)** Use the zero feature of the graphing calculator to find the positive $x$-intercept is approximately 0.3. So, the spider lands on the ground 0.3 s after it jumps.




**e)** Since neither time nor height can be negative, the domain is $\{t \mid 0 \le t \le 0.3, t \in R\}$ and the range is $\{h \mid 0 \le h \le 14.9, h \in R\}$.

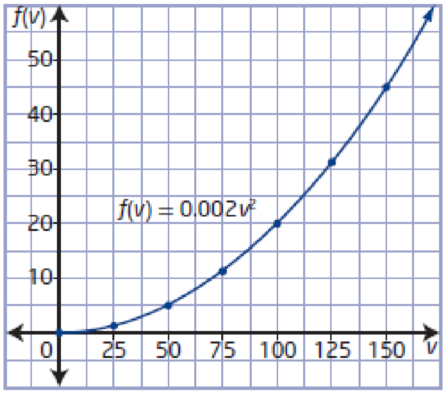
**f)** To determine the height of the spider 0.05 s after it jumps, substitute $t = 0.05$ into $h(t) = -490t^2 + 75t + 12$.
$h(0.05) = -490(0.05)^2 + 75(0.05) + 12$
$h(0.05) = 14.525$
The height of the spider 0.05 s after it jumps is approximately 14.5 cm.

**Section 3.2  Page 176  Question 13**

**a)** Answers may vary. Example: Since the speed of the vehicle cannot be negative, for a typical car travelling on a highway an appropriate domain for $f(v) = 0.002v^2$ is $\{v \mid 0 \le v \le 150, v \in \mathrm{R}\}$.

**b)**

| *v* | *f* |
|---|---|
| 0 | 0 |
| 25 | 1.25 |
| 50 | 5 |
| 75 | 11.25 |
| 100 | 20 |
| 125 | 31.25 |
| 150 | 45 |




**c)** The graph shows that the function is not linear because it is a curve. The table of values shows that the function is not linear because the values of *f* are not increasing at a constant rate for equal increments in the value of *v*.

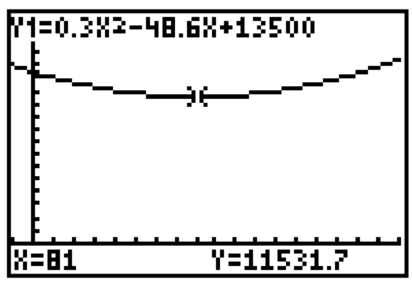
**d)** When the speed of the vehicle doubles, the drag force quadruples. For example, for $v = 50$, $f = 5$and for $v = 100$, $f = 20$.

**e)** Answers may vary. Example: The driver can use this information to understand why fuel consumption increases in windy conditions.

**Section 3.2  Page 177  Question 14**

**a)** Graph $C(n) = 0.3n^2 - 48.6n + 13\ 500$ using a graphing calculator with window settings of *x*: [–10, 200, 10] and *y*: [–2000, 15000, 1000].

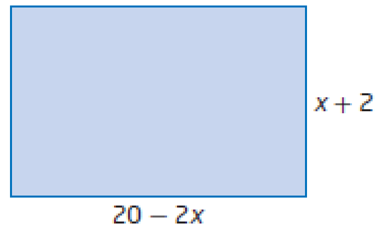
vertex: (81, 11531.7)
axis of symmetry: $x = 81$
opens upward
minimum value: 11531.7
domain: $\{x \mid x \ge 0, x \in \mathrm{R}\}$
range: $\{y \mid y \ge 11531.7, y \in \mathrm{R}\}$
*x*-intercepts: none
*y*-intercept: 13 500




**b)** Answers may vary. Example: The vertex represents the minimum cost of $11 531.70 to produce 81 000 units. Since there are no *n*-intercepts, the cost of production is always greater than zero. The *C*-intercept represents the base production cost of $13 500. The domain represents thousands of units produced, and the range represents the cost to produce those units.

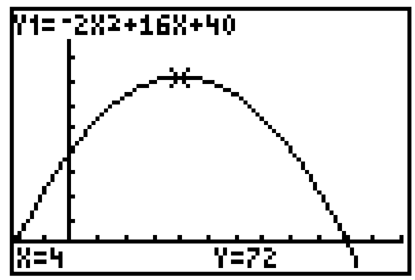
## Section 3.2 Page 177 Question 15

**a)** Write a function for the area of a rectangle using $A = lw$.




$A = (20 - 2x)(x + 2)$
$A = 20x + 40 - 2x^2 - 4x$
$A = -2x^2 + 16x + 40$
The function fits the definition of a quadratic since it is a polynomial of degree two.

**b)** Graph $A = -2x^2 + 16x + 40$ using a graphing calculator with window settings of $x$: [–2, 12, 1] and $y$: [–10, 100, 10].




**c)** The portion of the graph above the $x$-axis represents the possible areas for the rectangle. So, the $x$-intercepts give the possible range of $x$-values that produce those areas.

**d)** The vertex gives the maximum area and the $x$-value for which it occurs.

**e)** The domain is $\{x \mid -2 \le x \le 10, x \in \mathrm{R}\}$ and the range is $\{A \mid 0 \le A \le 72, A \in \mathrm{R}\}$. The domain represents the values for $x$ that will produce dimensions of a rectangle. The range represents the possible values of the area of the rectangle.

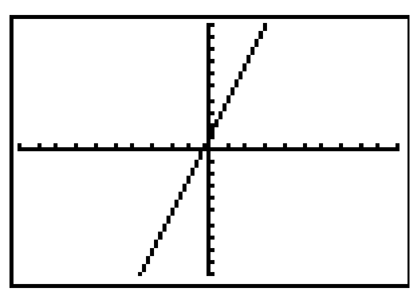
**f)** The function has a maximum value, 72, and a minimum value, 0.

**g)** If the domain is $\{x \mid x \in \mathrm{R}\}$, then the function has no minimum.

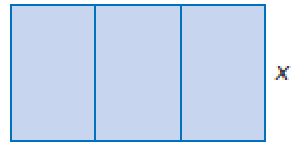
## Section 3.2 Page 177 Question 16

Method 1: Expand $f(x) = 4x^2 - 3x + 2x(3 - 2x) + 1$ to determine whether it represents a quadratic function.
$f(x) = 4x^2 - 3x + 6x - 4x^2 + 1$
$f(x) = 3x + 1$
The function is not quadratic, since it is of degree one.

Method 2: Graph $f(x) = 4x^2 - 3x + 2x(3 - 2x) + 1$ using a graphing calculator with window settings of $x$: [–10, 10, 1] and $y$: [–10, 10, 1]. The function is not quadratic, since the graph is linear.



**Section 3.2 Page 177 Question 17**




**a)** There is 280 m of fencing. Determine an expression for the length of the entire enclosure.

Length = $\dfrac{280-4x}{2}$

Length = $140 - 2x$

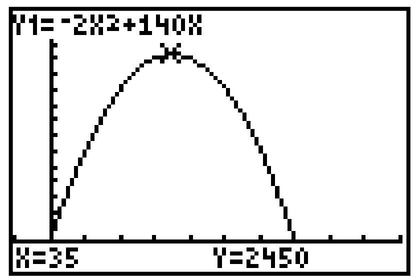
Write a function for the area of the entire rectangle using $A = lw$.

$A = (140 - 2x)x$

$A = 140x - 2x^2$

$A = -2x^2 + 140x$

The function fits the definition of a quadratic since it is a polynomial of degree two.




**b)** Graph $A = -2x^2 + 140x$ using a graphing calculator with window settings of $x$: [–10, 100, 10] and $y$: [–400, 3000, 200].

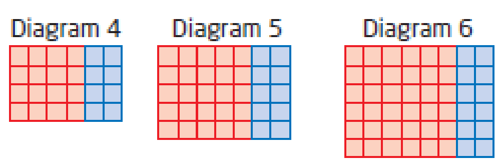
**c)** Use the maximum feature to determine the coordinates of the vertex are (35, 2450). This represents the maximum area of the entire enclosure and the width at which it occurs.

**d)** The domain is $\{x \mid 0 \le x \le 2450, x \in \mathrm{R}\}$ and the range is $\{A \mid 0 \le A \le 2450, A \in \mathrm{R}\}$. The domain represents the possible values for the width. The range represents the possible values for the area of the entire enclosure.

**e)** The function has a maximum value and a minimum value, since it represents possible areas.

**f)** Answers may vary. Example: Assume that all the fencing is used.

**Section 3.2 Page 177 Question 18**




**a)** Continuing the pattern, the next three diagrams are shown. Since the area of each small square is
1 square unit, the total area of a diagram is equal to the number of small squares it contains.

Diagram 4: 4(6) = 24 square units

Diagram 5: 5(7) = 35 square units

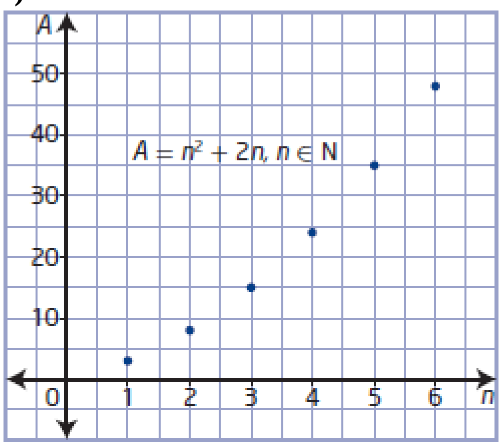
Diagram 6: 6(8) = 48 square units

**b)** A function to model the total area, $A$, of each diagram in terms of the diagram number, $n$ is $A = n^2 + 2n$.

**c)** The function is part b) is quadratic since it is a polynomial of degree two. This is also represented by the increasing side lengths of the entire red area, which is a square.

**d)** The domain is $\{n \mid n \geq 1, n \in \mathrm{N}\}$. Since the values of $n$ belong to the set of natural numbers, the function is discrete.

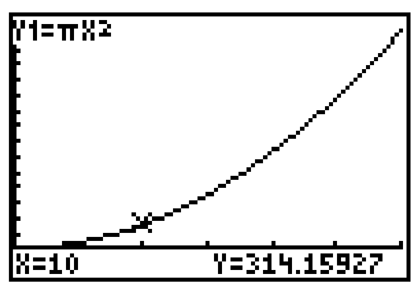
**e)**




**Section 3.2 Page 178 Question 19**

**a)** A function for the area, $A$, of a circle, in terms of its radius, $r$, is $A = \pi r^2$.

**b)** Since both $A$ are $r$ cannot be negative, the domain is $\{r \mid r \geq 0, r \in \mathrm{R}\}$ and the range is $\{A \mid A \geq 0, A \in \mathrm{R}\}$.




**c)** Graph $A = \pi r^2$ using a graphing calculator with window settings of $x$: [0, 30, 5] and $y$: [–300, 3000, 200].

**d)** The $x$-intercept and the $y$-intercept occur at (0, 0). They represent the minimum values of the radius and the area.

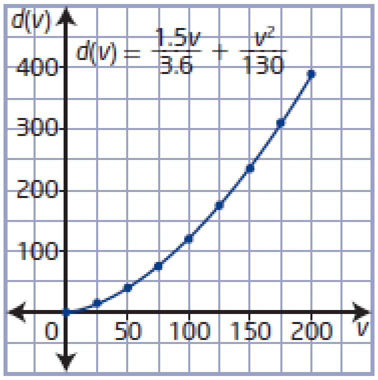
**e)** Answers may vary. Example: The axis of symmetry would be $x = 0$, but the graph shows only the right half of the parabola.

**Section 3.2 Page 178 Question 20**

**a)** Substitute $t = 1.5$ into $d(t) = \frac{vt}{3.6} + \frac{v^2}{130}$. Then, a function to model the stopping distance, $d$, for the vehicle and driver as a function of the pre-braking speed, $v$ is $d(v) = \frac{1.5v}{3.6} + \frac{v^2}{130}$.

**b)**

| v | d |
|---|---|
| 0 | 0 |
| 25 | 15 |
| 50 | 40 |
| 75 | 75 |
| 100 | 119 |
| 125 | 172 |
| 150 | 236 |
| 175 | 308 |
| 200 | 391 |




**c)** When the speed of the vehicle doubles, the stopping distance more than doubles. For example, when $v = 50$, $d = 40$, but when $v = 100$, $d = 119$.

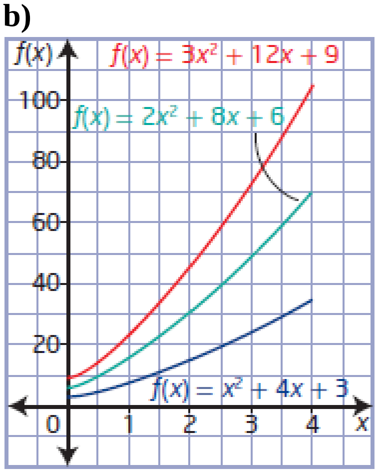
**d)** Answers may vary. The argument aimed at convincing drivers to slow down should include the results from part c) and a graph.

**Section 3.2 Page 178 Question 21**

**a)** A set of functions for part of the family defined by $f(x) = k(x^2 + 4x + 3)$ if $k = 1, 2, 3$ is

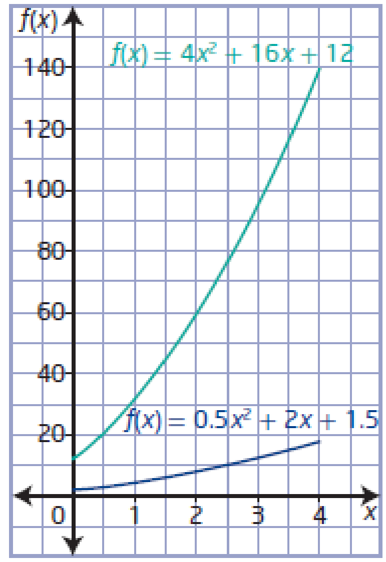
| | | |
|---|---|---|
| $f(x) = 1(x^2 + 4x + 3)$ | $f(x) = 2(x^2 + 4x + 3)$ | $f(x) = 3(x^2 + 4x + 3)$ |
| $f(x) = x^2 + 4x + 3$ | $f(x) = 2x^2 + 8x + 6$ | $f(x) = 3x^2 + 12x + 9$ |

**b)**

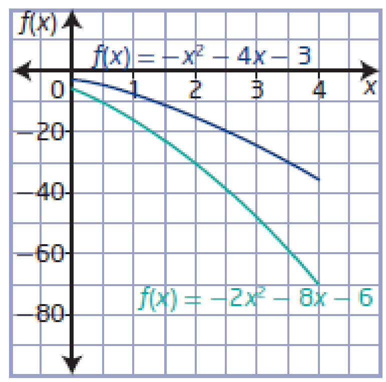


**c)** Answers may vary. Example: The graphs have similar shapes. They increasingly curve upward. The values of $y$ on each graph for the same value of $x$ are multiples of each other.

**d)** Answers may vary. Example: For $k = 4$, the graph would lie above the graph for $k = 3$, starting at (0, 4) with $y$-values that are 4 times those of the graph for $k = 1$. For $k = 0.5$, the graph would lie below the graph for $k = 1$, starting at (0, 0.5) with $y$-values that are 0.5 times those of the graph for $k = 1$.




**e)** Answers may vary. Example: The graphs for negative values of $k$ will be reflections in the $x$-axis of the corresponding positive $k$-value graph.




**f)** The graph for $k = 0$ is line defined by $f(x) = 0$. The graph is a line on the $x$-axis.

**g)** Answers may vary. Example: Each member of the family of functions for $f(x) = k(x^2 + 4x + 3)$ has $y$-values that are multiples of the $y$-values of $f(x) = x^2 + 4x + 3$ for each corresponding $x$-value.

**Section 3.2 Page 178 Question 22**

Answers may vary. Example: The $a$ in the quadratic function $f(x) = ax^2 + bx + c$ may appear to define the 'steepness' of the graph. For example, as positive $a$-values increase, the parabola rises faster and faster. However, it is not the slope since the graph is a curve with ever changing slope.

**Section 3.2 Page 178 Question 23**

**a)** Substitute the coordinates of the given point (–2, 1) into $f(x) = -x^2 + bx + 11$ to find $b$.

$1 = -(-2)^2 + b(-2) + 11$
$1 = -4 - 2b + 11$
$2b = 6$
$b = 3$

**b)** Substitute the coordinates of the given points (–1, 6) and then (2, 3) into $f(x) = 2x^2 + bx + c$. solve the resulting linear system of equations to find $b$.

For (–1, 6),
$6 = 2(-1)^2 + b(-1) + c$
$6 = 2 - b + c$
$4 = -b + c$ ①

For (2, 3),
$3 = 2(2)^2 + b(2) + c$
$3 = 8 + 2b + c$
$-5 = 2b + c$ ②

Solve the system by elimination.

$4 = -b + c$ ①
$\underline{-5 = 2b + c}$ ②
$9 = -3b$ ① – ②
$b = -3$

Then, substitute $b = -3$ into ① to find $c$.

$4 = -(-3) + c$
$c = 1$

**Section 3.2 Page 179 Question 24**

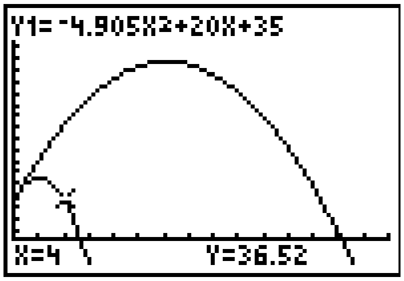
**a)** Scenario 1: For an object launched from an initial height of 35 m above ground with an initial vertical velocity of 20 m/s, substitute $h_0 = 35$ and $v_0 = 20$ into $h(t) = -0.5gt^2 + v_0t + h_0$.
For the situation on Earth use $g = 9.81$ and for the moon use $g = 1.63$.
Earth: $h(t) = -4.905t^2 + 20t + 35$
Moon: $h(t) = -0.815t^2 + 20t + 35$

Scenario 2: For a flare that is shot into the air with an initial velocity of 800 ft/s from ground level, substitute $h_0 = 0$ and $v_0 = 800$ into $h(t) = -0.5gt^2 + v_0t + h_0$. For the situation on Earth use $g = 32$ and for the moon use $g = 5.38$.
Earth: $h(t) = -16t^2 + 800t$
Moon: $h(t) = -2.69t^2 + 800t$

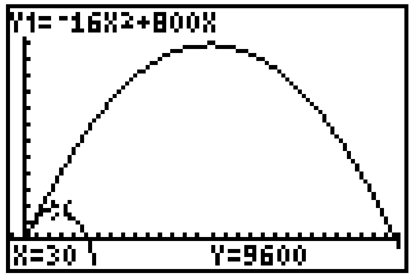
Scenario 3: For a rock that breaks loose from the top of a 100-m-high cliff and starts to fall straight down, substitute $h_0 = 100$ and $v_0 = 0$ into $h(t) = -0.5gt^2 + v_0t + h_0$. For the situation on Earth use $g = 9.81$ and for the moon use $g = 1.63$.
Earth: $h(t) = -4.905t^2 + 100$
Moon: $h(t) = -0.815t^2 + 100$

**b)** Use a graphing calculator to graph each pair of functions from part a).

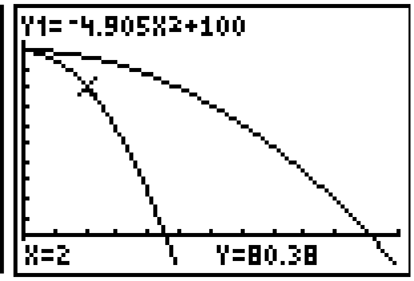
Scenario 1:




Scenario 2:




Scenario 3:




**c)** Answers may vary. Example: In scenario 1, the graphs have the same *y*-intercept of 35 but different maximum values (about 55 and 158) and *x*-intercepts (about 5 and 26). In scenario 2, the graphs have the same *y*-intercept of 0 and one *x*-intercept of 0 but different maximum values (10 000 and about 59480) and second *x*-intercepts (50 and about 297). In scenario 3, the graphs have the same *y*-intercept and maximum value of 100 but different *x*-intercepts (about 5 and 11).

**d)** Answers may vary. Example: Every projectile on the moon had a higher trajectory and stayed in the air longer than when on Earth.

**Section 3.2 Page 179 Question 25**

Answers may vary. Examples:

**a)** For a quadratic function with vertex at $(m, n)$ and a *y*-intercept of $r$, the function is of the form $y = a(x - m)^2 + n$ and a point on the graph is $(0, r)$. Use symmetry to find another point on the parabola. Assume that the given point is to the left of axis of symmetry, $x = m$. Then, the horizontal distance to the point is $m - 0$, or $m$. The *x*-coordinate of the corresponding point on the other side of the axis of symmetry is $m + m$, or $2m$. So, another point on the parabola is $(2m, r)$.

**b)** For a quadratic function with axis of symmetry of $x = j$ and passing through the point $(4j, k)$, the function is of the form $y = a(x - j)^2 + q$. Use symmetry to find another point on the parabola. Assume that the given point is to the right of axis of symmetry. Then, the horizontal distance to the point is $4j - j$, or $3j$. The *x*-coordinate of the corresponding point on the other side of the axis of symmetry is $j - 3j$, or $-2j$. So, another point on the parabola is $(-2j, k)$.

**c)** For a quadratic function with a range of $y \geq d$ and $x$-intercepts of $s$ and $t$, the function is of the form $y = a(x - p)^2 + d$ and two points on the graph are $(s, 0)$ and $(t, 0)$.

The $x$-coordinate of the vertex is halfway between the $x$-intercepts, or $\frac{s+t}{2}$.

The $y$-coordinate of the vertex is given by the minimum value of the range, or $d$.

So, the vertex is at $\left(\frac{s+t}{2}, d\right)$.

**Section 3.2 Page 179 Question 26**

Answers may vary. Example:

• The range, direction of opening, and location of vertex are connected. The direction of opening and the $y$-coordinate of the vertex can be determined from the range. If the range is $\{y \mid y \geq q, y \in \mathrm{R}\}$, the parabola opens upward and the $y$-coordinate of the vertex is $q$, which is the minimum value. If the range is $\{y \mid y \leq q, y \in \mathrm{R}\}$, the parabola opens downward and the $y$-coordinate of the vertex is $q$, which is the maximum value.

• The axis of symmetry and the location of the vertex are connected. The axis of symmetry gives the $x$-coordinate of the vertex.

• The location of the vertex and the number of $x$-intercepts are connected. If the vertex is above the $x$-axis and the graph opens upward, there will be no $x$-intercepts. However, if it opens downward, there will be two $x$-intercepts. If the vertex is below the $x$-axis and the graph opens upward, there will be two $x$-intercepts. However, if it opens downward, there will be no $x$-intercepts. If the vertex is on the $x$-axis, there will be only one $x$-intercept.

**Section 3.2 Page 179 Question 27**

Answers may vary. Example:

**Step 2** The values of $a$ and $b$ do not affect the location of the $y$-intercept. The $y$-intercept is determined by the value of $c$.

**Step 3** The axis of symmetry is affected by the values of $a$ and $b$. Recall that the equation of the axis of symmetry is given by $x = \frac{-b}{2a}$. So, if $b$ is constant and the value of $a$ increases, the value of the axis of symmetry decreases. If $a$ is constant and the value of $b$ increases, the value of the axis of symmetry increases.

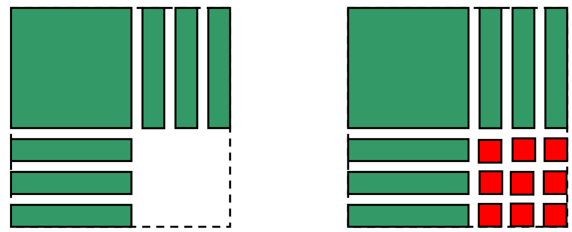
**Step 4** Increasing the value of $a$ increases the steepness of the graph. The values of $b$ and $c$ have no effect.

**Step 5** Changing the values of $a$, $b$, and $c$ affects the position of the vertex and the direction of opening.

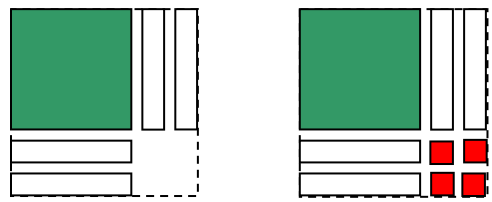
## Section 3.3 Completing the Square

### Section 3.3 Page 192 Question 1

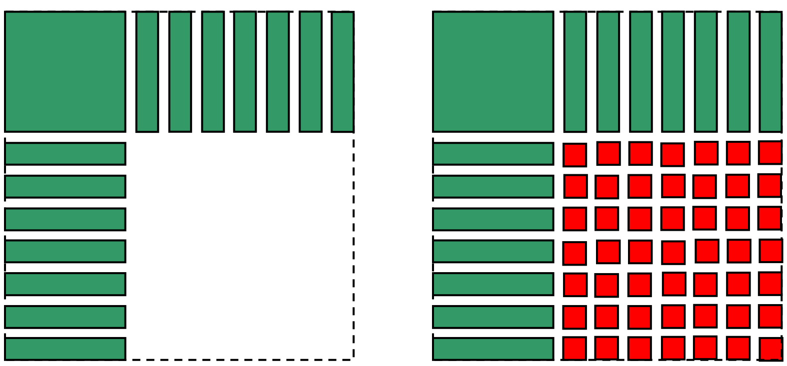
**a)** Select algebra tiles to model $x^2 + 6x + c$.
To complete the square, add nine unit tiles.
So, $c = 9$.
The equivalent binomial square is $(x + 3)^2$.



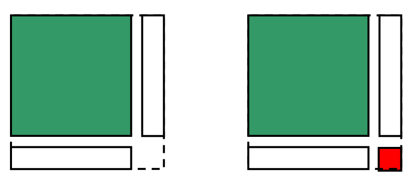
**b)** Select algebra tiles to model $x^2 - 4x + c$. To complete the square, add four unit tiles. So, $c = 4$.
The equivalent binomial square is $(x - 2)^2$.



**c)** Select algebra tiles to model $x^2 + 14x + c$. To complete the square, add 49 unit tiles. So, $c = 49$. The equivalent binomial square is $(x + 7)^2$.



**d)** Select algebra tiles to model $x^2 - 2x + c$. To complete the square, add one unit tile. So, $c = 1$.
The equivalent binomial square is $(x - 1)^2$.



### Section 3.3 Page 192 Question 2

**a)** Complete the square to write $y = x^2 + 8x$ in vertex form.
$y = x^2 + 8x$
$y = x^2 + 8x + 16 - 16$
$y = (x^2 + 8x + 16) - 16$
$y = (x + 4)^2 - 16$
The vertex of the function is $(-4, -16)$.

**b)** Complete the square to write $y = x^2 - 18x - 59$ in vertex form.
$y = x^2 - 18x - 59$
$y = (x^2 - 18x) - 59$
$y = (x^2 - 18x + 81 - 81) - 59$
$y = (x^2 - 18x + 81) - 81 - 59$
$y = (x - 9)^2 - 81 - 59$
$y = (x - 9)^2 - 140$
The vertex of the function is (9, –140).

**c)** Complete the square to write $y = x^2 - 10x + 31$ in vertex form.
$y = x^2 - 10x + 31$
$y = (x^2 - 10x) + 31$
$y = (x^2 - 10x + 25 - 25) + 31$
$y = (x^2 - 10x + 25) - 25 + 31$
$y = (x - 5)^2 - 25 + 31$
$y = (x - 5)^2 + 6$
The vertex of the function is (5, 6).

**d)** Complete the square to write $y = x^2 + 32x - 120$ in vertex form.
$y = x^2 + 32x - 120$
$y = (x^2 + 32x) - 120$
$y = (x^2 + 32x + 256 - 256) - 120$
$y = (x^2 + 32x + 256) - 256 - 120$
$y = (x + 16)^2 - 256 - 120$
$y = (x + 16)^2 - 376$
The vertex of the function is (–16, –376).

**Section 3.3 Page 193 Question 3**

**a)** Complete the square to write $y = 2x^2 - 12x$ in the form $y = a(x - p)^2 + q$.
$y = 2x^2 - 12x$
$y = 2(x^2 - 6x)$
$y = 2(x^2 - 6x + 9 - 9)$
$y = 2[(x^2 - 6x + 9) - 9]$
$y = 2[(x - 3)^2 - 9]$
$y = 2(x - 3)^2 - 18$
Expand $y = 2(x - 3)^2 - 18$ to verify the two forms are equivalent.
$y = 2(x - 3)^2 - 18$
$y = 2(x^2 - 6x + 9) - 18$
$y = 2x^2 - 12x + 18 - 18$
$y = 2x^2 - 12x$

**b)** Complete the square to write $y = 6x^2 + 24x + 17$ in the form $y = a(x - p)^2 + q$.
$y = 6x^2 + 24x + 17$
$y = 6(x^2 + 4x) + 17$
$y = 6(x^2 + 4x + 4 - 4) + 17$
$y = 6[(x^2 + 4x + 4) - 4] + 17$
$y = 6[(x + 2)^2 - 4] + 17$
$y = 6(x + 2)^2 - 24 + 17$
$y = 6(x + 2)^2 - 7$
Expand $y = 6(x + 2)^2 - 7$ to verify the two forms are equivalent.
$y = 6(x + 2)^2 - 7$
$y = 6(x^2 + 4x + 4) - 7$
$y = 6x^2 + 24x + 24 - 7$
$y = 6x^2 + 24x + 17$

**c)** Complete the square to write $y = 10x^2 - 160x + 80$ in the form $y = a(x - p)^2 + q$.
$y = 10x^2 - 160x + 80$
$y = 10(x^2 - 16x) + 80$
$y = 10(x^2 - 16x + 64 - 64) + 80$
$y = 10[(x^2 - 16x + 64) - 64] + 80$
$y = 10[(x - 8)^2 - 64] + 80$
$y = 10(x - 8)^2 - 640 + 80$
$y = 10(x - 8)^2 - 560$
Expand $y = 10(x - 8)^2 - 560$ to verify the two forms are equivalent.
$y = 10(x - 8)^2 - 560$
$y = 10(x^2 - 16x + 64) - 560$
$y = 10x^2 - 160x + 640 - 560$
$y = 10x^2 - 160x + 80$

**d)** Complete the square to write $y = 3x^2 + 42x - 96$ in the form $y = a(x - p)^2 + q$.
$y = 3x^2 + 42x - 96$
$y = 3(x^2 + 14x) - 96$
$y = 3(x^2 + 14x + 49 - 49) - 96$
$y = 3[(x^2 + 14x + 49) - 49] - 96$
$y = 3[(x + 7)^2 - 49] - 96$
$y = 3(x + 7)^2 - 147 - 96$
$y = 3(x + 7)^2 - 243$
Expand $y = 3(x + 7)^2 - 243$ to verify the two forms are equivalent.
$y = 3(x + 7)^2 - 243$
$y = 3(x^2 + 14x + 49) - 243$
$y = 3x^2 + 42x + 147 - 243$
$y = 3x^2 + 42x - 96$

**Section 3.3 Page 193 Question 4**

**a)** Covert $f(x) = –4x^2 + 16x$ to vertex form.
$f(x) = –4x^2 + 16x$
$f(x) = –4(x^2 – 4x)$
$f(x) = –4(x^2 – 4x + 4 – 4)$
$f(x) = –4[(x^2 – 4x + 4) – 4]$
$f(x) = –4[(x – 2)^2 – 4]$
$f(x) = –4(x – 2)^2 + 16$
Expand $f(x) = –4(x – 2)^2 + 16$ to verify the two forms are equivalent.
$f(x) = –4(x – 2)^2 + 16$
$f(x) = –4(x^2 – 4x + 4) + 16$
$f(x) = –4x^2 + 16x – 16 + 16$
$f(x) = –4x^2 + 16x$

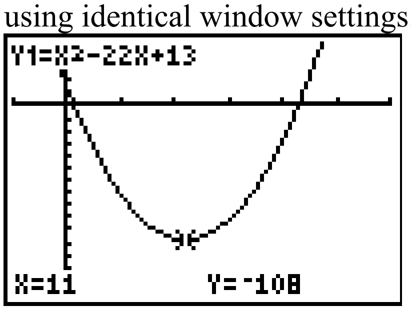
**b)** Covert $f(x) = –20x^2 – 400x – 243$ to vertex form.
$f(x) = –20x^2 – 400x – 243$
$f(x) = –20(x^2 + 20x) – 243$
$f(x) = –20(x^2 + 20x + 100 – 100) – 243$
$f(x) = –20[(x^2 + 20x + 100) – 100] – 243$
$f(x) = –20[(x + 10)^2 – 100] – 243$
$f(x) = –20(x + 10)^2 + 2000 – 243$
$f(x) = –20(x + 10)^2 + 1757$
Expand $f(x) = –20(x + 10)^2 + 1757$ to verify the two forms are equivalent.
$f(x) = –20(x + 10)^2 + 1757$
$f(x) = –20(x^2 + 20x + 100) + 1757$
$f(x) = –20x^2 – 400x – 2000 + 1757$
$f(x) = –20x^2 – 400x – 243$

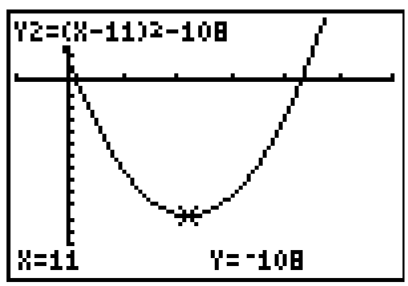
**c)** Covert $f(x) = –x^2 – 42x + 500$ to vertex form.
$f(x) = –x^2 – 42x + 500$
$f(x) = –(x^2 + 42x) + 500$
$f(x) = –(x^2 + 42x + 441 – 441) + 500$
$f(x) = –[(x^2 + 42x + 441) – 441] + 500$
$f(x) = –[(x + 21)^2 – 441] + 500$
$f(x) = –(x + 21)^2 + 441 + 500$
$f(x) = –(x + 21)^2 + 941$
Expand $f(x) = –(x + 21)^2 + 941$ to verify the two forms are equivalent.
$f(x) = –(x + 21)^2 + 941$
$f(x) = –(x^2 + 42x + 441) + 941$
$f(x) = –x^2 – 42x + 500$

**d)** Covert $f(x) = –7x^2 + 182x – 70$ to vertex form.
$f(x) = –7x^2 + 182x – 70$
$f(x) = –7(x^2 – 26x) – 70$
$f(x) = –7(x^2 – 26x + 169 – 169) – 70$
$f(x) = –7[(x^2 – 26x + 169) – 169] – 70$
$f(x) = –7[(x – 13)^2 – 169] – 70$
$f(x) = –7(x – 13)^2 + 1183 – 70$
$f(x) = –7(x – 13)^2 + 1113$
Expand $f(x) = –7(x – 13)^2 + 1113$ to verify the two forms are equivalent.
$f(x) = –7(x – 13)^2 + 1113$
$f(x) = –7(x^2 – 26x + 169) + 1113$
$f(x) = –7x^2 + 182x – 1183 + 1113$
$f(x) = –7x^2 + 182x – 70$

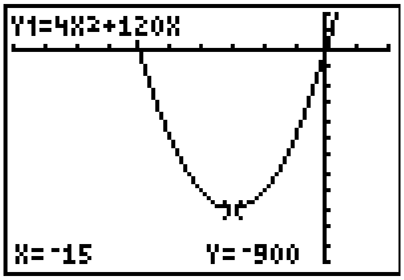
**Section 3.3 Page 193 Question 5**

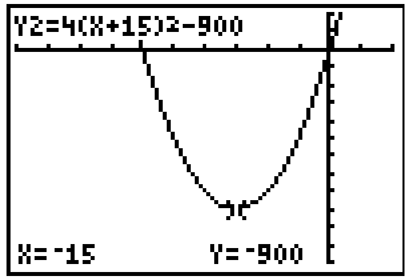
**a)** Verify that $y = x^2 – 22x + 13$ and $y = (x – 11)^2 – 108$ represent the same function.
Algebraically: Expand $y = (x – 11)^2 – 108$ and compare to $y = x^2 – 22x + 13$.
$y = (x – 11)^2 – 108$
$y = x^2 – 22x + 121 – 108$
$y = x^2 – 22x + 13$
Graphically: Use a graphing calculator to graph both functions together or separately using identical window settings.




**b)** Verify that $y = 4x^2 + 120x$ and $y = 4(x + 15)^2 – 900$ represent the same function, algebraically and graphically.
Algebraically:
$y = 4(x + 15)^2 – 900$
$y = 4(x^2 + 30x + 225) – 900$
$y = 4x^2 + 120x + 900 – 900$
$y = 4x^2 + 120x$
Graphically:



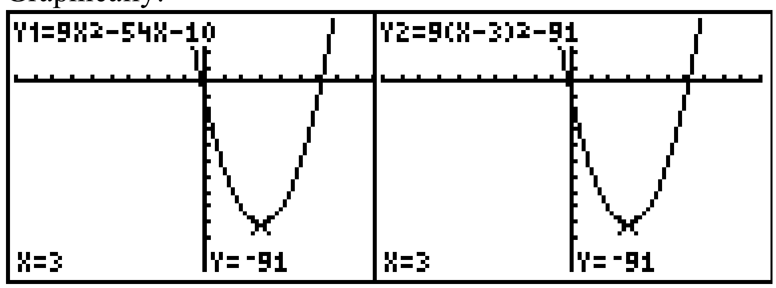




**c)** Verify that $y = 9x^2 - 54x - 10$ and $y = 9(x - 3)^2 - 91$ represent the same function, algebraically and graphically.

Algebraically:

$y = 9(x - 3)^2 - 91$
$y = 9(x^2 - 6x + 9) - 91$
$y = 9x^2 - 54x + 81 - 91$
$y = 9x^2 - 54x - 10$

Graphically:

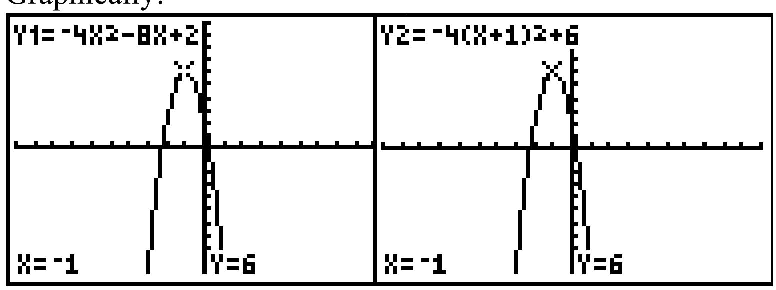



**d)** Verify that $y = -4x^2 - 8x + 2$ and $y = -4(x + 1)^2 + 6$ represent the same function, algebraically and graphically.

Algebraically:

$y = -4(x + 1)^2 + 6$
$y = -4(x^2 + 2x + 1) + 6$
$y = -4x^2 - 8x - 4 + 6$
$y = -4x^2 - 8x + 2$

Graphically:




**Section 3.3 Page 193 Question 6**

**a)** Complete the square to determine the maximum or minimum value of $y = x^2 + 6x - 2$.

$y = x^2 + 6x - 2$
$y = (x^2 + 6x + 9 - 9) - 2$
$y = (x^2 + 6x + 9) - 9 - 2$
$y = (x + 3)^2 - 11$

Since $a > 0$, the graph has a minimum value of –11 when $x = -3$.

**b)** Complete the square to determine the maximum or minimum value of $y = 3x^2 - 12x + 1$.

$y = 3x^2 - 12x + 1$
$y = 3(x^2 - 4x) + 1$
$y = 3(x^2 - 4x + 4 - 4) + 1$
$y = 3[(x^2 - 4x + 4) - 4] + 1$
$y = 3[(x - 2)^2 - 4] + 1$
$y = 3(x - 2)^2 - 12 + 1$
$y = 3(x - 2)^2 - 11$

Since $a > 0$, the graph has a minimum value of –11 when $x = 2$.

**c)** Complete the square to determine the maximum or minimum value of $y = x^2 + 6x - 2$.

$y = -x^2 - 10x$
$y = -(x^2 + 10x)$
$y = -(x^2 + 10x + 25 - 25)$
$y = -[(x^2 + 10x + 25) - 25]$
$y = -[(x + 5)^2 - 25]$
$y = -(x + 5)^2 + 25$

Since $a < 0$, the graph has a maximum value of 25 when $x = -5$.

**d)** Complete the square to determine the maximum or minimum value of $y = -2x^2 + 8x - 3$.

$y = -2(x^2 - 4x) - 3$
$y = -2(x^2 - 4x + 4 - 4) - 3$
$y = -2[(x^2 - 4x + 4) - 4] - 3$
$y = -2[(x - 2)^2 - 4] - 3$
$y = -2(x - 2)^2 + 8 - 3$
$y = -2(x - 2)^2 + 5$

Since $a < 0$, the graph has a maximum value of 5 when $x = 2$.

**Section 3.3 Page 193 Question 7**

**a)** Complete the square to determine the maximum or minimum value.

$f(x) = x^2 + 5x + 3$
$f(x) = (x^2 + 5x) + 3$
$f(x) = (x^2 + 5x + 6.25 - 6.25) + 3$
$f(x) = (x^2 + 5x + 6.25) - 6.25 + 3$
$f(x) = (x + 2.5)^2 - 6.25 + 3$
$f(x) = (x + 2.5)^2 - 3.25$

Since $a > 0$, the graph has a minimum value of –3.25.

**b)** Complete the square to determine the maximum or minimum value.
$f(x) = 2x^2 - 2x + 1$
$f(x) = 2(x^2 - x) + 1$
$f(x) = 2(x^2 - x + 0.25 - 0.25) + 1$
$f(x) = 2[(x^2 - x + 0.25) - 0.25] + 1$
$f(x) = 2[(x - 0.5)^2 - 0.25] + 1$
$f(x) = 2(x - 0.5)^2 - 0.5 + 1$
$f(x) = 2(x - 0.5)^2 + 0.5$
Since $a > 0$, the graph has a minimum value of 0.5.

**c)** Complete the square to determine the maximum or minimum value.
$f(x) = -0.5x^2 + 10x - 3$
$f(x) = -0.5(x^2 - 20x) - 3$
$f(x) = -0.5(x^2 - 20x + 100 - 100) - 3$
$f(x) = -0.5[(x^2 - 20x + 100) - 100] - 3$
$f(x) = -0.5[(x - 10)^2 - 100] - 3$
$f(x) = -0.5(x - 10)^2 + 50 - 3$
$f(x) = -0.5(x - 10)^2 + 47$
Since $a < 0$, the graph has a maximum value of 47.

**d)** Complete the square to determine the maximum or minimum value.
$f(x) = 3x^2 - 4.8x$
$f(x) = 3(x^2 - 1.6x)$
$f(x) = 3(x^2 - 1.6x + 0.64 - 0.64)$
$f(x) = 3[(x^2 - 1.6x + 0.64) - 0.64]$
$f(x) = 3[(x - 0.8)^2 - 0.64]$
$f(x) = 3(x - 0.8)^2 - 1.92$
Since $a > 0$, the graph has a minimum value of –1.92.

**e)** Complete the square to determine the maximum or minimum value.
$f(x) = -0.2x^2 + 3.4x + 4.5$
$f(x) = -0.2(x^2 - 17x) + 4.5$
$f(x) = -0.2(x^2 - 17x + 72.25 - 72.25) + 4.5$
$f(x) = -0.2[(x^2 - 17x + 72.25) - 72.25] + 4.5$
$f(x) = -0.2[(x - 8.5)^2 - 72.25] + 4.5$
$f(x) = -0.2(x - 8.5)^2 + 14.45 + 4.5$
$f(x) = -0.2(x - 8.5)^2 + 18.95$
Since $a < 0$, the graph has a maximum value of 18.95.

**f)** Complete the square to determine the maximum or minimum value.
$f(x) = –2x^2 + 5.8x – 3$
$f(x) = –2(x^2 – 2.9x) – 3$
$f(x) = –2(x^2 – 2.9x + 2.1025 – 2.1025) – 3$
$f(x) = –2[(x^2 – 2.9x + 2.1025) – 2.1025] – 3$
$f(x) = –2[(x – 1.45)^2 – 2.1025] – 3$
$f(x) = –2(x – 1.45)^2 + 4.205 – 3$
$f(x) = –2(x – 1.45)^2 + 1.205$
Since $a < 0$, the graph has a maximum value of 1.205.

**Section 3.3 Page 193 Question 8**

**a)** Convert $y = x^2 + \frac{3}{2}x – 7$ to vertex form.

$$y = x^2 + \frac{3}{2}x - 7$$

$$y = \left(x^2 + \frac{3}{2}x\right) - 7$$

$$y = \left(x^2 + \frac{3}{2}x + \frac{9}{16} - \frac{9}{16}\right) - 7$$

$$y = \left(x^2 + \frac{3}{2}x + \frac{9}{16}\right) - \frac{9}{16} - 7$$

$$y = \left(x + \frac{3}{4}\right)^2 - \frac{121}{16}$$

**b)** Convert $y = –x^2 – \frac{3}{8}x$ to vertex form.

$$y = -x^2 + \frac{3}{8}x$$

$$y = -\left(x^2 - \frac{3}{8}x\right)$$

$$y = -\left(x^2 - \frac{3}{8}x + \frac{9}{256} - \frac{9}{256}\right)$$

$$y = -\left[\left(x^2 + \frac{3}{8}x + \frac{9}{256}\right) - \frac{9}{256}\right]$$

$$y = -\left[\left(x + \frac{3}{16}\right)^2 - \frac{9}{256}\right]$$

$$y = -\left(x + \frac{3}{16}\right)^2 + \frac{9}{256}$$

**c)** Convert $y = 2x^2 - \frac{5}{6}x + 1$ to vertex form.

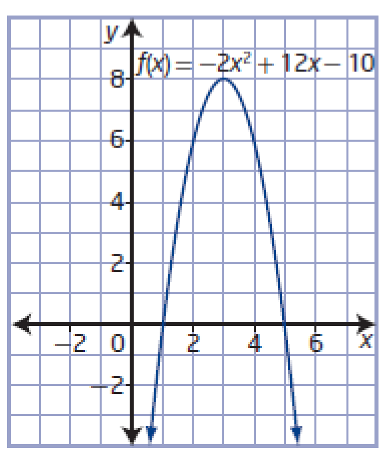
$$y = 2x^2 - \frac{5}{6}x + 1$$
$$y = 2\left(x^2 - \frac{5}{12}x\right) + 1$$
$$y = 2\left(x^2 - \frac{5}{12}x + \frac{25}{576} - \frac{25}{576}\right) + 1$$
$$y = 2\left[\left(x^2 - \frac{5}{12}x + \frac{25}{576}\right) - \frac{25}{576}\right] + 1$$
$$y = 2\left[\left(x - \frac{5}{24}\right)^2 - \frac{25}{576}\right] + 1$$
$$y = 2\left(x - \frac{5}{24}\right)^2 - \frac{25}{288} + 1$$
$$y = 2\left(x - \frac{5}{24}\right)^2 + \frac{263}{288}$$

**Section 3.3 Page 193 Question 9**

**a)** Convert $f(x) = -2x^2 + 12x - 10$ to vertex form.

$f(x) = -2x^2 + 12x - 10$
$f(x) = -2(x^2 - 6x) - 10$
$f(x) = -2(x^2 - 6x + 9 - 9) - 10$
$f(x) = -2[(x^2 - 6x + 9) - 9] - 10$
$f(x) = -2[(x - 3)^2 - 9] - 10$
$f(x) = -2(x - 3)^2 + 18 - 10$
$f(x) = -2(x - 3)^2 + 8$

**b)** Answers may vary. Example: The graph shows the vertex at (3, 8), which agrees with vertex form found in part a).



**Section 3.3 Page 193 Question 10**

**a)** Complete the square to determine the maximum or minimum value.
$y = -4x^2 + 20x + 37$
$y = -4(x^2 - 5x) + 37$
$y = -4(x^2 - 5x + 6.25 - 6.25) + 37$
$y = -4[(x^2 - 5x + 6.25) - 6.25] + 37$
$y = -4[(x - 2.5)^2 - 6.25] + 37$
$y = -4(x - 2.5)^2 + 25 + 37$
$y = -4(x - 2.5)^2 + 62$
Since $a < 0$, the graph has a maximum value of 62.
The domain is $\{x \mid x \in R\}$ and the range is $\{y \mid y \le 62, y \in R\}$.

**b)** To find the maximum or minimum value of a quadratic function in standard form without making a table of values of graphing, complete the square to change the function to vertex form. The value of $p$ is the maximum, since the parabola opens downward ($a < 0$). This also means that the range is $\{y \mid y \le p, y \in R\}$. The domain is the set of real numbers since this quadratic function does not represent a real-life situation that typically has restrictions.

**Section 3.3 Page 193 Question 11**

Complete the square to determine the vertex of $f(x) = 12x^2 - 78x + 126$.
$f(x) = 12x^2 - 78x + 126$
$f(x) = 12(x^2 - 6.5x) + 126$
$f(x) = 12(x^2 - 6.5x + 10.5625 - 10.5625) + 126$
$f(x) = 12[(x^2 - 6.5x + 10.5625) - 10.5625] + 126$
$f(x) = 12[(x - 3.25)^2 - 10.5625] + 126$
$f(x) = 12(x - 3.25)^2 - 126.75 + 126$
$f(x) = 12(x - 3.25)^2 - 0.75$
The vertex is at (–3.25, –0.75).

**Section 3.3 Page 194 Question 12**

**a)** Identify and correct errors in the given example of completing the square:
$y = x^2 + 8x + 30$
$y = (x^2 + 4x + 4) + 30$
$y = (x + 2)^2 + 30$
There are errors in line 2 of the solution. The coefficient of the $x$-term was divided by 2. Then, the square of half of this incorrect coefficient was added and subtracted. The coefficient of the $x$-term should be 8 in line 2. Then, add and subtract 16.
$y = x^2 + 8x + 30$
$y = (x^2 + 8x + 16 - 16) + 30$
$y = (x + 4)^2 + 14$

**b)** Identify and correct errors in the given example of completing the square:

$f(x) = 2x^2 – 9x – 55$
$f(x) = 2(x^2 – 4.5x + 20.25 – 20.25) – 55$
$f(x) = 2[(x^2 – 4.5x + 20.25) – 20.25] – 55$
$f(x) = 2[(x – 4.5)^2 – 20.25] – 55$
$f(x) = 2(x – 4.5)^2 – 40.5 – 55$
$f(x) = (x – 4.5)^2 – 95.5$

There is an error in line 2 of the solution. The square of the coefficient of the $x$-term was added and subtracted. There is also an error in line 6. The factor of 2 is missing. The square of half the coefficient of the $x$-term, 5.0625, should be added and subtracted in line 2. Insert the 2 factor in line 6.

$f(x) = 2x^2 – 9x – 55$
$f(x) = 2(x^2 – 4.5x + 5.0625 – 5.0625) – 55$
$f(x) = 2[(x^2 – 4.5x + 5.0625) – 5.0625] – 55$
$f(x) = 2[(x – 2.25)^2 – 5.0625] – 55$
$f(x) = 2(x – 2.25)^2 – 10.125 – 55$
$f(x) = 2(x – 2.25)^2 – 65.125$

**c)** Identify and correct errors in the given example of completing the square:

$y = 8x^2 + 16x – 13$
$y = 8(x^2 + 2x) – 13$
$y = 8(x^2 + 2x + 4 – 4) – 13$
$y = 8[(x^2 + 2x + 4) – 4] – 13$
$y = 8[(x + 2)^2 – 4] – 13$
$y = 8(x + 2)^2 – 32 – 13$
$y = 8(x + 2)^2 – 45$

There is an error in line 3 of the solution. The square of the coefficient of the $x$-term was added and subtracted. Should add and subtract the square of half the coefficient of the $x$-term, 1.

$y = 8x^2 + 16x – 13$
$y = 8(x^2 + 2x) – 13$
$y = 8(x^2 + 2x + 1 – 1) – 13$
$y = 8[(x^2 + 2x + 1) – 1] – 13$
$y = 8[(x + 1)^2 – 1] – 13$
$y = 8(x + 1)^2 – 8 – 13$
$y = 8(x + 1)^2 – 21$

**d)** Identify and correct errors in the given example of completing the square:

$f(x) = –3x^2 – 6x$
$f(x) = –3(x^2 – 6x – 9 + 9)$
$f(x) = –3[(x^2 – 6x – 9) + 9]$
$f(x) = –3[(x – 3)^2 + 9]$
$f(x) = –3(x – 3)^2 + 27$

There are errors in line 2. –3 was not factored out of the coefficient of the $x$-term. The square of half the coefficient of the $x$-term was subtracted then added. There is also an

error in line 5. –3 was not distributed properly. In line 2, the coefficient of the $x$-term should be 2, then add and subtract 1.
$f(x) = -3x^2 - 6x$
$f(x) = -3(x^2 + 2x + 1 - 1)$
$f(x) = -3[(x^2 + 2x + 1) - 1]$
$f(x) = -3[(x + 1)^2 - 1]$
$f(x) = -3(x + 1)^2 + 3$

**Section 3.3 Page 194 Question 13**

Complete the square to determine the vertex of $C(n) = 75n^2 - 1800n + 60\ 000$.
$C(n) = 75n^2 - 1800n + 60\ 000$
$C(n) = -75(n^2 + 24n) + 60\ 000$
$C(n) = -75(n^2 + 24n + 144 - 144) + 60\ 000$
$C(n) = -75[(n^2 + 24n + 144) - 144] + 60\ 000$
$C(n) = -75[(n + 12)^2 - 144] + 60\ 000$
$C(n) = -75(n + 12)^2 + 10\ 800 + 60\ 000$
$C(n) = -75(n + 12)^2 + 70\ 800$
The business should produce 12 000 items to minimize their costs at $70 800.

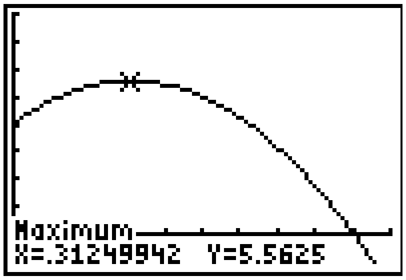
**Section 3.3 Page 194 Question 14**

Complete the square to determine the vertex of $h(t) = -5t^2 + 10t + 4$.
$h(t) = -5t^2 + 10t + 4$
$h(t) = -5(t^2 - 2t) + 4$
$h(t) = -5(t^2 - 2t + 1 - 1) + 4$
$h(t) = -5[(t^2 - 2t + 1) - 1] + 4$
$h(t) = -5[(t - 1)^2 - 1] + 4$
$h(t) = -5(t - 1)^2 + 5 + 4$
$h(t) = -5(t - 1)^2 + 9$
The maximum height of the gymnast on each jump is 9 m.

**Section 3.3 Page 194 Question 15**

**a)** Complete the square to determine the vertex of $h(t) = -16t^2 + 10t + 4$.
$h(t) = -16t^2 + 10t + 4$
$h(t) = -16(t^2 - 0.625t) + 4$
$h(t) = -16(t^2 - 0.625t + 0.3125^2 - 0.3125^2) + 4$
$h(t) = -16[(t^2 - 0.625t + 0.3125^2) - 0.3125^2] + 4$
$h(t) = -16[(t - 0.3125)^2 - 0.3125^2] + 4$
$h(t) = -16(t - 0.3125)^2 + 1.5625 + 4$
$h(t) = -16(t - 0.3125)^2 + 5.5625$
The maximum height of the arrow is 5.5625 ft 0.3125 s after it was fired.

**b)** Answers may vary. Examples:
Verify by graphing $h(t) = -16t^2 + 10t + 4$ and finding the vertex.




Alternatively, use $t = \frac{-b}{2a}$ to find the $t$-coordinate of the vertex.

$t = \frac{-10}{2(-16)}$

$t = 0.3125$

Substitute $t = 0.3125$ into $h(t) = -16t^2 + 10t + 4$ to find the $h$-coordinate of the vertex.

$h(0.3125) = -16(0.3125)^2 + 10(0.3125) + 4$

$h(0.3125) = 5.5625$

The vertex is located at (0.3125, 5.5625).

**Section 3.3 Page 194 Question 16**

**a)** *Austin's solution:*
$y = -6x^2 + 72x - 20$
$y = -6(x^2 + 12x) - 20$
$y = -6(x^2 + 12x + 36 - 36) - 20$
$y = -6[(x^2 + 12x + 36) - 36] - 20$
$y = -6[(x + 6) - 36] - 20$
$y = -6(x + 6) + 216 - 20$
$y = -6(x + 6) + 196$
There is an error in line 2. –6 was not correctly factored out of the coefficient of the $x$-term. The coefficient of the $x$-term should be –12. In line 5, the expression $(x + 6)$ was not squared.
$y = -6x^2 + 72x - 20$
$y = -6(x^2 - 12x) - 20$
$y = -6(x^2 - 12x + 36 - 36) - 20$
$y = -6[(x^2 - 12x + 36) - 36] - 20$
$y = -6[(x - 6)^2 - 36] - 20$
$y = -6(x - 6)^2 + 216 - 20$
$y = -6(x - 6)^2 + 196$

*Yuri's solution:*
$y = -6x^2 + 72x - 20$
$y = -6(x^2 - 12x) - 20$
$y = -6(x^2 - 12x + 36 - 36) - 20$
$y = -6[(x^2 - 12x + 36) - 36] - 20$

$y = -6[(x - 6)^2 - 36] - 20$
$y = -6(x - 6)^2 - 216 - 20$
$y = -6(x - 6)^2 + 236$
There is an error in line 6. –6 was not correctly distributed: –6(–36) = 216. Then, the corrected function is $y = -6(x - 6)^2 + 196$.

**b)** Answers may vary. Example: To verify an answer, either work backward to show the functions are equivalent or use technology to show the graphs of the functions are identical. In either case, Austin and Yuri would have found that the functions and graphs were not equivalent.

**Section 3.3 Page 195 Question 17**

Complete the square to determine the vertex of $d(x) = 0.03125x^2 - 1.5x$.
$d(x) = 0.03125x^2 - 1.5x$
$d(x) = 0.03125(x^2 - 48x)$
$d(x) = 0.03125(x^2 - 48x + 576 - 576)$
$d(x) = 0.03125[(x^2 - 48x + 576) - 576]$
$d(x) = 0.03125[(x - 24)^2 - 576]$
$d(x) = 0.03125(x - 24)^2 - 18$
The dish has a depth of 18 cm at its centre.

**Section 3.3 Page 195 Question 18**

**a)** Write a function to model this situation.
Let $n$ represent the number of price decreases. The new price is \$70 minus the number of price decreases times \$1, or $70 - n$.
The new number of tickets sold is 2000 plus the number of price decreases times 50, or $2000 + 50n$.
Let $R$ represent the expected revenue, in dollars.
Revenue = (price)(number of sessions)
$R = (70 - n)(2000 + 50n)$
$R = 140\ 000 + 1500n - 50n^2$
$R = -50n^2 + 1500n + 140\ 000$
Complete the square to find the vertex.
$R = -50n^2 + 1500n + 140\ 000$
$R = -50(n^2 - 30n) + 140\ 000$
$R = -50(n^2 - 30n + 225 - 225) + 140\ 000$
$R = -50[(n^2 - 30n + 225) - 225] + 140\ 000$
$R = -50[(n - 15)^2 - 225] + 140\ 000$
$R = -50(n - 15)^2 + 11\ 250 + 140\ 000$
$R = -50(n - 15)^2 + 151\ 250$
The maximum revenue the promoter can expect is \$151 250 when the ticket price is 70 – 15, or \$55.

**b)** The promoter can expect to sell 151 250 ÷ 55, or 2750 tickets.

**c)** Answers may vary. Example: Assume that the decrease in ticket prices determines the same increase in ticket sales continually, as indicated by the survey.

**Section 3.3 Page 195 Question 19**

**a)** Write a function to model this situation.
Let $n$ represent the number of price increases. The new price is \$360 plus the number of price increases times \$10, or $360 + 10n$.
The new number of bikes sold is 280 minus the number of price increases times 5, or $280 – 5n$.
Let $R$ represent the expected revenue, in dollars.
Revenue = (price)(number of sessions)
$R = (360 + 10n)(280 – 5n)$
$R = 100\ 800 + 1000n – 50n^2$
$R = –50n^2 + 1000n + 100\ 800$

**b)** Complete the square to find the vertex.
$R = –50n^2 + 1000n + 100\ 800$
$R = –50(n^2 – 20n) + 100\ 800$
$R = –50(n^2 – 20n + 100 – 100) + 100\ 800$
$R = –50[(n^2 – 20n + 100) – 100] + 100\ 800$
$R = –50[(n – 10)^2 – 100] + 100\ 800$
$R = –50(n – 10)^2 + 5000 + 100\ 800$
$R = –50(n – 10)^2 + 105\ 800$
The maximum revenue the manager can expect is \$105 800 when a bike sells for 360 + 10(10), or \$460.

**c)** Answers may vary. Example: Assume that the manager’s predictions regarding price and number of bikes sold holds true.

**Section 3.3 Page 196 Question 20**

**a)** Let $n$ represent the number of additional rows planted. The new number of rows is 30 plus the number of additional rows times 1, or $30 + n$.
The new yield per row, in grams, is 4000 minus the number of additional rows times 100, or $4000 – 100n$. In kilograms this becomes $4 – 0.1n$.
Let $P$ represent the peas produced, in kilograms.
Peas produced = (yield per row)(number of rows)
$P = (4 – 0.1n)(30 + n)$
$P = 120 + n – 0.1n^2$
$P = –0.1n^2 + n + 120$

**b)** Complete the square to find the vertex.
$P = -0.1n^2 + n + 120$
$P = -0.1(n^2 - 10n) + 120$
$P = -0.1(n^2 - 10n + 25 - 25) + 120$
$P = -0.1[(n^2 - 10n + 25) - 25] + 120$
$P = -0.1[(n - 5)^2 - 25] + 120$
$P = -0.1(n - 5)^2 + 2.5 + 120$
$P = -0.1(n - 5)^2 + 122.5$
The maximum pea production from the field is 122.5 kg when the number of rows planted is 30 + 5, or 35 rows.

**c)** Answers may vary. Example: Assume that the gardener's estimates regarding pea production and number of rows planted holds true.

**Section 3.3 Page 196 Question 21**

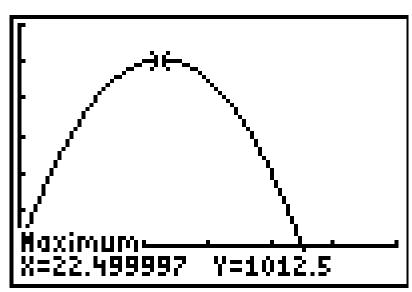
**a)** Answers may vary. Example: I predict the dimensions will be 22.5 m by 45 m.

**b)** Let $w$ represent the width of the pen. Let $l$ represent the length of the pen. Then, $l = 90 - 2w$.
Write a function for the area of a rectangle using $A = lw$.
$A = (90 - 2w)(w)$
$A = 90w - 2w^2$
$A = -2w^2 + 90w$

**c)** Complete the square to find the maximum area.
$A = -2w^2 + 90w$
$A = -2(w^2 - 45w)$
$A = -2(w^2 - 45w + 506.25 - 506.25)$
$A = -2[(w^2 - 45w + 506.25) - 506.25]$
$A = -2[(w - 22.5)^2 - 506.25]$
$A = -2(w - 22.5)^2 + 1012.5$

The maximum possible area is 1012.5 $m^2$.

**d)** Answers may vary. Examples:
Verify by graphing $A = -2w^2 + 90w$ and finding the vertex.




The vertex is located at (22.5, 1012.5). So, the maximum possible area is 1012.5 $m^2$.

Alternatively, use $w = \frac{-b}{2a}$ to find the $w$-coordinate of the vertex.

$$w = \frac{-90}{2(-2)}$$

$w = 22.5$

Substitute $w = 22.5$ into $A = -2w^2 + 90w$ to find the $A$-coordinate of the vertex.

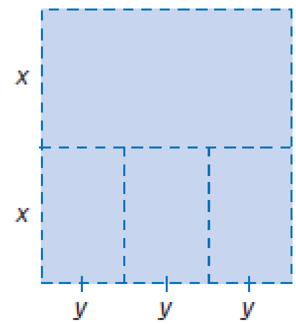
$A = -2(22.5)^2 + 90(22.5)$

$A = 1012.5$

The vertex is located at (22.5, 1012.5). So, the maximum possible area is 1012.5 $m^2$. My prediction was correct.

**e)** Answers may vary. Example: Assume that all of the fencing is used.

**Section 3.3 Page 196 Question 22**




Write an expression for the total amount of fencing:

$900 = 6x + 9y$, or

$y = 100 - \frac{2}{3}x$.

Write a function for the overall area.

$A = 2x(3y)$

$$A = 2x\left[3\left(100 - \frac{2}{3}x\right)\right]$$

$A = 2x(300 - 2x)$

$A = 600x - 4x^2$

$A = -4x^2 + 600x$

Complete the square to find the maximum area.

$A = -4x^2 + 600x$

$A = -4(x^2 - 150x)$

$A = -4(x^2 - 150x + 5625 - 5625)$

$A = -4[(x^2 - 150x + 5625) - 5625]$

$A = -4[(x - 75)^2 - 5625]$

$A = -4(x - 75)^2 + 22\,500$

So, $x = 75$. Substitute into $y = 100 - \frac{2}{3}x$ to find $y$.

$$y = 100 - \frac{2}{3}(75)$$

$y = 50$

The measurements that will maximize the overall area are $x = 75$ m and $y = 50$ m.

**Section 3.3 Page 196 Question 23**

**a)** Let $x$ represent one number. Then, $29 - x$ represents the other number. Complete the square to find the maximum product of $y = x(29 - x)$.

$y = -x^2 + 29x$
$y = -(x^2 - 29x)$
$y = -(x^2 - 29x + 210.25 - 210.25)$
$y = -[(x^2 - 29x + 210.25) - 210.25]$
$y = -[(x - 14.5)^2 - 210.25]$
$y = -(x - 14.5)^2 + 210.25$

A maximum product of 210.25 occurs when one number is 14.5 and the other is 29 – 14.5, or 14.5.

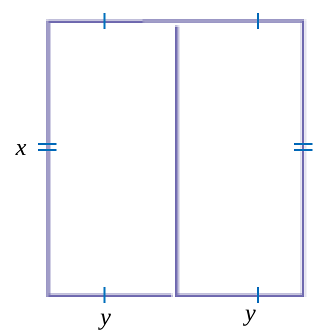
**b)** Let $x$ represent one number. Then, $x + 13$ represents the other number. Complete the square to find the minimum product of $y = x(x + 13)$.

$y = x^2 + 13x$
$y = x^2 + 13x + 42.25 - 42.25$
$y = (x^2 + 13x + 42.25) - 42.25$
$y = (x + 6.5)^2 - 42.25$

A minimum product of –42.25 occurs when one number is –6.5 and the other is –6.5 + 13, or 6.5.

**Section 3.3 Page 196 Question 24**

Let $x$ represent the length of the small rectangle. Let $y$ represent the width of the small rectangle. Write an expression for the total length of string:




$450 = 3x + 4y$, or $y = \dfrac{450 - 3x}{4}$.

Write a function for the overall area.

$A = x(2y)$
$A = x\left[2\left(\dfrac{450 - 3x}{4}\right)\right]$
$A = x(225 - 1.5x)$
$A = 225x - 1.5x^2$

Complete the square to find the maximum area.

$A = -1.5x^2 + 225x$
$A = -1.5(x^2 - 150x)$
$A = -1.5(x^2 - 150x + 5625 - 5625)$
$A = -1.5[(x^2 - 150x + 5625) - 5625]$
$A = -1.5[(x - 75)^2 - 5625]$
$A = -1.5(x - 75)^2 + 8437.5$

The maximum total area is 8437.5 $cm^2$.

**Section 3.3 Page 196 Question 25**

Write $f(x) = -\frac{3}{4}x^2 + \frac{9}{8}x + \frac{5}{16}$ in vertex form.

$$f(x) = -\frac{3}{4}x^2 + \frac{9}{8}x + \frac{5}{16}$$
$$f(x) = -\frac{3}{4}\left(x^2 - \frac{3}{2}x\right) + \frac{5}{16}$$
$$f(x) = -\frac{3}{4}\left(x^2 - \frac{3}{2}x + \frac{9}{16} - \frac{9}{16}\right) + \frac{5}{16}$$
$$f(x) = -\frac{3}{4}\left[\left(x^2 - \frac{3}{2}x + \frac{9}{16}\right) - \frac{9}{16}\right] + \frac{5}{16}$$
$$f(x) = -\frac{3}{4}\left[\left(x - \frac{3}{4}\right)^2 - \frac{9}{16}\right] + \frac{5}{16}$$
$$f(x) = -\frac{3}{4}\left(x - \frac{3}{4}\right)^2 + \frac{27}{64} + \frac{5}{16}$$
$$f(x) = -\frac{3}{4}\left(x - \frac{3}{4}\right)^2 + \frac{47}{64}$$

**Section 3.3 Page 196 Question 26**

**a)** Complete the square for $y = ax^2 + bx + c$.

$$y = ax^2 + bx + c$$
$$y = a\left(x^2 + \frac{b}{a}x\right) + c$$
$$y = a\left(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2\right) + c$$
$$y = a\left[\left(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2\right) - \left(\frac{b}{2a}\right)^2\right] + c$$
$$y = a\left[\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2}\right] + c$$
$$y = a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c$$
$$y = a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a}$$

**b)** The vertex is located at $\left(-\frac{b}{2a}, \frac{4ac - b^2}{4a}\right)$.

**c)** Answers may vary. Example: This formula can be used to find the vertex of any quadratic function without completing the square to change the function to vertex form.

**Section 3.3 Page 197 Question 27**

**a)** Use $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$ to determine the vertex of the function $f(x) = 2x^2 - 12x + 22$.

$$\frac{-b}{2a} = \frac{-(-12)}{2(2)}$$
$$\frac{-b}{2a} = 3$$
$$f\left(\frac{-b}{2a}\right) = f(3)$$
$$f\left(\frac{-b}{2a}\right) = 2(3)^2 - 12(3) + 22$$
$$f\left(\frac{-b}{2a}\right) = 18 - 36 + 22$$
$$f\left(\frac{-b}{2a}\right) = 4$$

The vertex of $f(x) = 2x^2 - 12x + 22$ is located at (3, 4).

**b)** Complete the square to find the vertex.

$f(x) = 2x^2 - 12x + 22$
$f(x) = 2(x^2 - 6x) + 22$
$f(x) = 2(x^2 - 6x + 9 - 9) + 22$
$f(x) = 2[(x^2 - 6x + 9) - 9] + 22$
$f(x) = 2[(x - 3)^2 - 9] + 22$
$f(x) = 2(x - 3)^2 - 18 + 22$
$f(x) = 2(x - 3)^2 + 4$

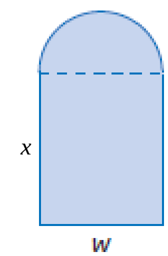
**c)** There is a relationship between the parameters *a*, *b*, and *c* in standard form and the parameters *a*, *p*, and *q* in vertex form.

$a = a$ $\qquad$ $p = \frac{-b}{2a}$ $\qquad$ $q = f\left(\frac{-b}{2a}\right)$ or $\frac{4ac - b^2}{4a}$

**Section 3.3 Page 197 Question 28**

**a)** Let $x$ represent the length of the rectangle.
Write an expression for the perimeter of the Norman window:

$6 = 2x + w + \frac{\pi w}{2}$, or $x = 3 - \frac{2+\pi}{4}w$.




Write a function for the overall area.

$$A = xw + \frac{\pi w^2}{8}$$

$$A = \left(3 - \frac{2+\pi}{4}w\right)w + \frac{\pi w^2}{8}$$

$$A = 3w - \frac{2+\pi}{4}w^2 + \frac{\pi w^2}{8}$$

$$A = \frac{-4-\pi}{8}w^2 + 3w$$

$$A = -\frac{4+\pi}{8}w^2 + 3w$$

**b)** Complete the square to find the vertex.

$$A = -\frac{4+\pi}{8}w^2 + 3w$$

$$A = -\frac{4+\pi}{8}\left(w^2 - \frac{24}{4+\pi}w\right)$$

$$A = -\frac{4+\pi}{8}\left(w^2 - \frac{24}{4+\pi}w + \left(\frac{12}{4+\pi}\right)^2 - \left(\frac{12}{4+\pi}\right)^2\right)$$

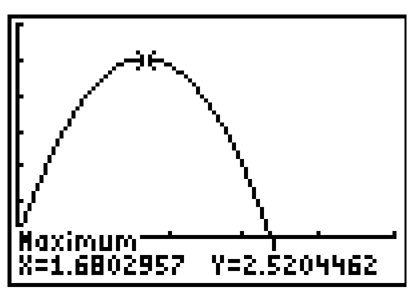
$$A = -\frac{4+\pi}{8}\left[\left(w^2 - \frac{24}{4+\pi}w + \left(\frac{12}{4+\pi}\right)^2\right) - \left(\frac{12}{4+\pi}\right)^2\right]$$

$$A = -\frac{4+\pi}{8}\left[\left(w - \frac{12}{4+\pi}\right)^2 - \frac{144}{(4+\pi)^2}\right]$$

$$A = -\frac{4+\pi}{8}\left(w - \frac{12}{4+\pi}\right)^2 + \frac{18}{4+\pi}$$

The maximum possible area is $\frac{18}{4+\pi}$, or approximately 2.52 m$^2$ when the width is $\frac{12}{4+\pi}$, or approximately 1.68 m.

**c)** Verify the answer to part b) with technology.



**d)** The diameter of the semicircle is $\frac{12}{4+\pi}$, or approximately 1.68 m.

Find the length of the rectangle when $w = \frac{12}{4+\pi}$.

$x = 3 - \frac{2+\pi}{4}w$

$x = 3 - \frac{2+\pi}{4}\left(\frac{12}{4+\pi}\right)$

$x = 3 - \frac{6+3\pi}{4+\pi}$

$x = \frac{6}{4+\pi}$

The length of the rectangle is $\frac{6}{4+\pi}$, or approximately 0.84 m.

Chosen scales for the scale diagram may vary.

The appearance of the window differs from expectations from in the diagram beside part a), as the "width" dimension is actually greater than the "length" dimension.

**Section 3.3 Page 197 Question 29**

Answers may vary. Examples:

**a)** An argument can be made for both forms, standard or vertex, for the quadratic function $f(x) = 4x^2 + 24$. If it is in standard form, then $a = 4$, $b = 0$, and $c = 24$. If it is vertex form, then $a = 4$, $p = 0$, and $q = 24$.

**b)** No, since it is already in this form.

**Section 3.3 Page 197 Question 30**

Find four errors in Martine's solution.

$y = -4x^2 + 24x + 5$
$y = -4(x^2 + 6x) + 5$
$y = -4(x^2 + 6x + 36 - 36) + 5$
$y = -4[(x^2 + 6x + 36) - 36] + 5$
$y = -4[(x + 6)^2 - 36] + 5$
$y = -4(x + 6)^2 - 216 + 5$
$y = -4(x + 6)^2 - 211$

The first error occurs in line 1. Martine did not correctly factor –4 from 24. The result should be –6, not +6.

The second error occurs in line 3. Martine did not add and subtract the square of half of the coefficient of the $x$-term. She should have added and subtracted 9, not 36.

The third error occurs in line 5. She factored $x^2 + 6x + 36$ as the square of a binomial, $(x + 6)^2$.
The last errors occur in line 6. She did not distribute –4 properly.
The correct solution is shown.
$y = –4x^2 + 24x + 5$
$y = –4(x^2 – 6x) + 5$
$y = –4(x^2 – 6x + 9 – 9) + 5$
$y = –4[(x^2 – 6x + 9) – 9] + 5$
$y = –4[(x – 3)^2 – 9] + 5$
$y = –4(x – 3)^2 + 36 + 5$
$y = –4(x – 3)^2 + 41$

**Section 3.3 Page 197 Question 31**

**a)** Let $n$ represent the number of price increases. The new T-shirt price is \$10 plus the number of price increases times \$1, or $10 + n$.
The new number of T-shirts sold each month is 100 minus the number of price increases times 5, or $100 – 5n$.
Let $R$ represent the expected monthly revenue, in dollars.
Revenue = (price)(number of sessions)
$R = (10 + n)(100 – 5n)$
$R = 1000 + 50n – 5n^2$
$R = –5n^2 + 50n + 1000$

**b)** By completing the square, you can determine the maximum monthly revenue and price to charge to produce that maximum revenue. You can also determine the number of T-shirts that must be sold to obtain the maximum revenue.

**c)** Answers may vary. Example: Assume that the market research for the price increase and number of T-shirts holds true for all sales.

**Chapter 3 Review**

**Chapter 3 Review Page 198 Question 1**

**a)** The graph of $f(x) = (x + 6)^2 – 14$ will have the same shape as the graph of $f(x) = x^2$, since $a = 1$. Since $p = –6$ and $q = –14$, this represents a horizontal translation of 6 units to the left and a vertical translation of 14 units down relative to the graph of $f(x) = x^2$.
The vertex is located at (–6, –14).
The equation of the axis of symmetry is $x = –6$.
The parabola opens upward.
The minimum value is –14.
The domain is $\{x \mid x \in R\}$ and the range is $\{y \mid y \geq –14, y \in R\}$.

**b)** The graph of $f(x) = -2x^2 + 19$ will have a shape that is narrower than the graph of $f(x) = x^2$ and be reflected in the $x$-axis, since $a < -1$. Since $p = 0$ and $q = 19$, this represents a vertical translation of 19 units up relative to the graph of $f(x) = x^2$.
The vertex is located at (0, 19).
The equation of the axis of symmetry is $x = 0$.
The parabola opens downward.
The maximum value is 19.
The domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \le 19, y \in \mathrm{R}\}$.

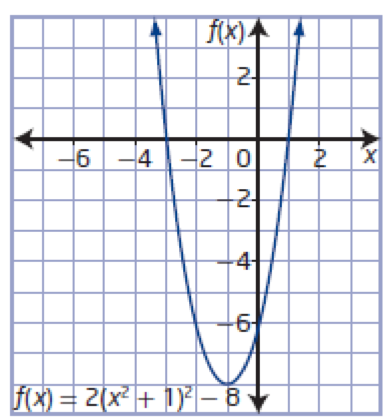
**c)** The graph of $f(x) = \frac{1}{5}(x - 10)^2 + 100$ will have a shape that is wider than the graph of $f(x) = x^2$, since $0 < a < 1$. Since $p = 10$ and $q = 100$, this represents a horizontal translation of 10 units to the right and a vertical translation of 100 units up relative to the graph of $f(x) = x^2$.
The vertex is located at (10, 100).
The equation of the axis of symmetry is $x = 10$.
The parabola opens upward.
The minimum value is 100.
The domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \ge 100, y \in \mathrm{R}\}$.

**d)** The graph of $f(x) = -6(x - 4)^2$ will have a shape that is narrower than the graph of $f(x) = x^2$ and be reflected in the $x$-axis, since $a < -1$. Since $p = 4$ and $q = 0$, this represents a horizontal translation of 4 units to the right relative to the graph of $f(x) = x^2$.
The vertex is located at (4, 0).
The equation of the axis of symmetry is $x = 4$.
The parabola opens downward.
The maximum value is 0.
The domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \le 0, y \in \mathrm{R}\}$.

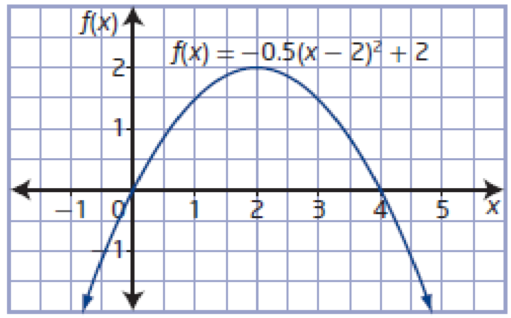
**Chapter 3 Review Page 198 Question 2**

**a)** For $f(x) = 2(x + 1)^2 - 8$, $a = 2$, $p = -1$, and $q = -8$.
To sketch the graph of $f(x) = 2(x + 1)^2 - 8$, transform the graph of $f(x) = x^2$ by
- multiplying the $y$-values by a factor of 2
- translating 1 unit to the left and 8 units down

vertex: (–1, –8)
axis of symmetry: $x = -1$
domain: $\{x \mid x \in \mathrm{R}\}$
range: $\{y \mid y \ge -8, y \in \mathrm{R}\}$
$x$-intercepts: –3 and 1
$y$-intercept: –6



**b)** For $f(x) = –0.5(x – 2)^2 + 2$, $a = –0.5$, $p = 2$, and $q = 2$.
To sketch the graph of $f(x) = –0.5(x – 2)^2 + 2$, transform the graph of $f(x) = x^2$ by
• multiplying the $y$-values by a factor of 0.5
• reflecting in the $x$-axis
• translating 2 units to the right and 2 units up
vertex: (2, 2)
axis of symmetry: $x = 2$
domain: $\{x \mid x \in R\}$
range: $\{y \mid y \leq 2, y \in R\}$
$x$-intercepts: 0 and 4
$y$-intercept: 0




**Chapter 3 Review Page 198 Question 3**

**a)** For $y = –3(x – 5)^2 + 20$, $a = –3$, $p = 5$, and $q = 20$.
The vertex is located at (5, 20), which is above the $x$-axis. The graph opens downward, since $a < 0$. So, there are two $x$-intercepts.

**b)** Since the range is $\{y \mid y \geq 0, y \in R\}$, the vertex is located on the $x$-axis. So, there is one $x$-intercept.

**c)** For $y = 9 + 3x^2$, $a = 3$, $p = 0$, and $q = 9$.
The vertex is located at (0, 9), which is above the $x$-axis. The graph opens upward, since $a > 0$. So, there are no $x$-intercepts.

**d)** Given a vertex at (–4, –6), the parabola either has no $x$-intercepts if it opens downward or two $x$-intercepts if it opens upward.

**Chapter 3 Review Page 198 Question 4**

**a)** vertex at (0, 0), passing through the point (20, −150)
Since $p = 0$ and $q = 0$, the function is of the form $y = ax^2$.
Substitute the coordinates of the given point to find $a$.
$–150 = a(20)^2$
$–150 = 400a$
$a = -\frac{3}{8}$

The quadratic function in vertex form with the given characteristics is $y = -\frac{3}{8}x^2$.

**b)** vertex at (8, 0), passing through the point (2, 54)
Since $p = 8$ and $q = 0$, the function is of the form $y = a(x - 8)^2$.
Substitute the coordinates of the given point to find $a$.

$54 = a(2 - 8)^2$
$54 = 36a$
$a = \frac{3}{2}$

The quadratic function in vertex form with the given characteristics is $y = \frac{3}{2}(x - 8)^2$.

**c)** minimum value of 12 at $x = –4$ and $y$-intercept of 60
Since $p = –4$ and $q = 12$, the function is of the form $y = a(x + 4)^2 + 12$.
Substitute the coordinates of the $y$-intercept to find $a$.

$60 = a(0 + 4)^2 + 12$
$48 = 16a$
$a = 3$

The quadratic function in vertex form with the given characteristics is $y = 3(x + 4)^2 + 12$.

**d)** $x$-intercepts of 2 and 7 and maximum value of 25
The $x$-coordinate is halfway between the $x$-intercepts. So, $p = 4.5$.
Since $p = 4.5$ and $q = 25$, the function is of the form $y = a(x – 4.5)^2 + 25$.
Substitute the coordinates of one of the $x$-intercepts to find $a$.

$0 = a(2 – 4.5)^2 + 25$
$–25 = 6.25a$
$a = –4$

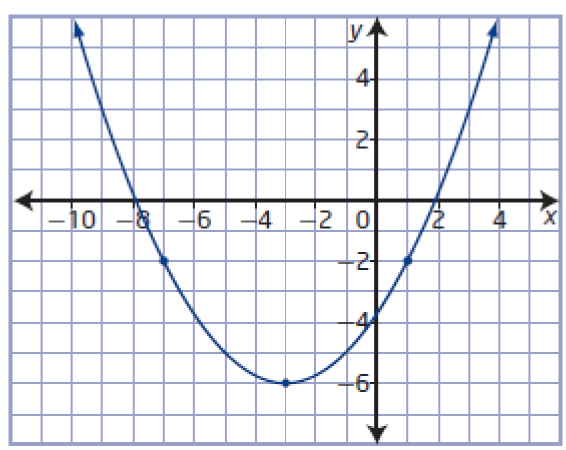
The quadratic function in vertex form with the given characteristics is
$y = –4(x – 4.5)^2 + 25$.

**Chapter 3 Review Page 198 Question 5**

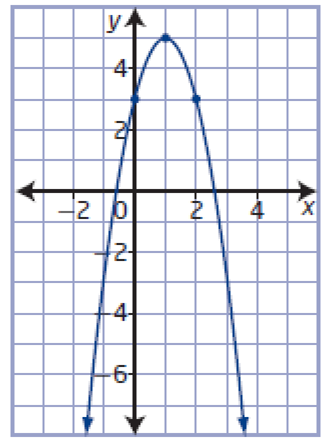
**a)** Since the vertex is located at (–3, –4), $p = –3$ and $q = –6$.
So, the function is of the form $y = a(x + 3)^2 – 6$. Substitute (1, –2) and solve for $a$.

$–2 = a(1 + 3)^2 – 6$
$4 = 16a$
$a = \frac{1}{4}$

The quadratic function in vertex form is
$y = \frac{1}{4}(x + 3)^2 – 6$.



**b)** Since the vertex is located at (1, 5), $p = 1$ and $q = 5$.
So, the function is of the form $y = a(x - 1)^2 + 5$. Substitute (0, 3) and solve for $a$.

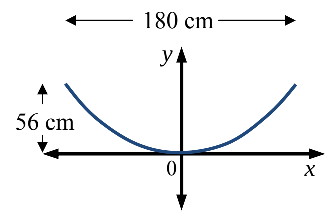


$3 = a(0 - 1)^2 + 5$
$3 = a + 5$
$a = -2$

The quadratic function in vertex form is $y = -2(x - 1)^2 + 5$.

**Chapter 3 Review  Page 198  Question 6**

Answers may vary. Example:




Choose the location of the origin to be the lowest point in the centre of the mirror. Let $x$ and $y$ represent the horizontal and vertical distances from the low point of the mirror, respectively.
Then, the vertex is at (0, 0) and the quadratic function is of the form $y = ax^2$. From the diagram, another point on the parabola is (90, 56).
Use the coordinates of this point to find $a$.

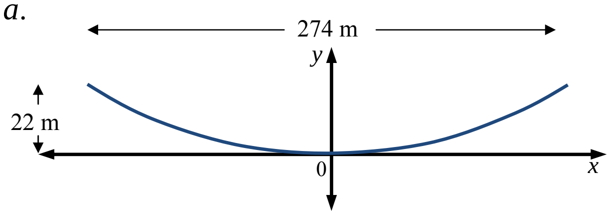
$56 = a(90)^2$
$56 = 8100a$
$a = \frac{14}{2025}$

A quadratic function that represents the cross-sectional shape is $y = \frac{14}{2025}x^2$.

**Chapter 3 Review  Page 199  Question 7**

**a) i)** The location of the origin is the lowest point in the centre of the cables.
Let $x$ and $y$ represent the horizontal and vertical distances from the low point of the cables, respectively. Then, the vertex is at (0, 0) and the quadratic function is of the form $y = ax^2$. From the diagram, another point on the parabola is (137, 22).
Use the coordinates of this point to find $a$.

$22 = a(137)^2$
$22 = 18\,769a$
$a = \frac{22}{18\,769}$




A quadratic function that represents the shape of the cables is $y = \frac{22}{18\,769}x^2$.

**ii)** The location of the origin is the point on the water's surface directly below the minimum point of the cables. Then the vertex is at (0, 30).
Let $x$ and $y$ represent the horizontal and vertical distances, respectively. The quadratic function is of the form $y = ax^2 + 30$. Another point on the parabola is (137, 52). Use the coordinates of this point to find $a$.

$52 = a(137)^2 + 30$

$22 = 18\,769a$

$a = \frac{22}{18\,769}$

A quadratic function that represents the shape of the cables is $y = \frac{22}{18\,769}x^2 + 30$.

**iii)** The location of the origin is the base of the tower on the left. Then the vertex is at (137, 30).
Let $x$ and $y$ represent the horizontal and vertical distances, respectively. The quadratic function is of the form $y = a(x - 137)^2 + 30$. Another point on the parabola is (0, 52). Use the coordinates of this point to find $a$.

$52 = a(0 - 137)^2 + 30$

$22 = 18\,769a$

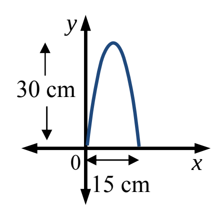
$a = \frac{22}{18\,769}$

A quadratic function that represents the shape of the cables is $y = \frac{22}{18\,769}(x - 137)^2 + 30$.

**b)** Answers may vary. Example: The function will change as the seasons change with the heat or cold changing the length of the cable.

**Chapter 3 Review    Page 199    Question 8**

The location of the origin is at the point from which the flea jumped. Let $x$ and $y$ represent the horizontal and vertical distances, respectively. Then, the vertex is at (7.5, 30) and the quadratic function is of the form $y = a(x - 7.5)^2 + 30$. From the diagram, another point on the parabola is (0, 0).
Use the coordinates of this point to find $a$.




$0 = a(0 - 7.5)^2 + 30$

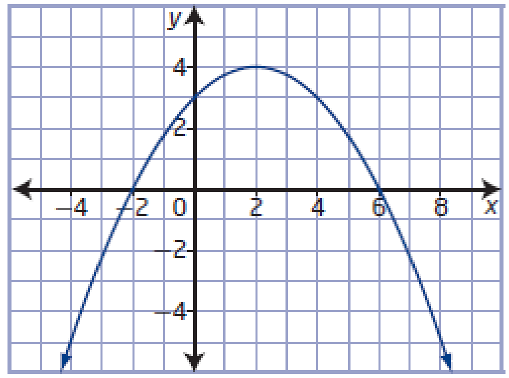
$-30 = 56.25a$

$a = -\frac{8}{15}$

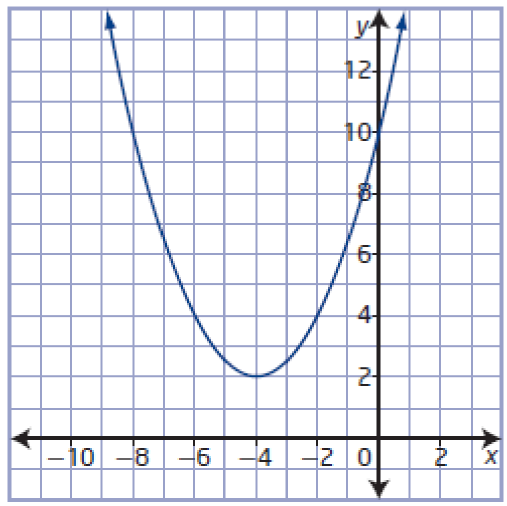
A quadratic function that represents the path of the flea is $y = -\frac{8}{15}(x - 7.5)^2 + 30$.

**Chapter 3 Review Page 199 Question 9**

**a)** The coordinates of the vertex are (2, 4).
The equation of the axis of symmetry is $x = 2$.
The graph has a maximum value of 4, since the parabola opens downward.
The domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \le 4, y \in \mathrm{R}\}$.
The $x$-intercepts are –2 and 6, and the $y$-intercept is 3.




**b)** The coordinates of the vertex are (–4, 2).
The equation of the axis of symmetry is $x = -4$.
The graph has a minimum value of 2, since the parabola opens upward.
The domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \ge 2, y \in \mathrm{R}\}$.
There are no $x$-intercepts, and the $y$-intercept is 10.




**Chapter 3 Review Page 199 Question 10**

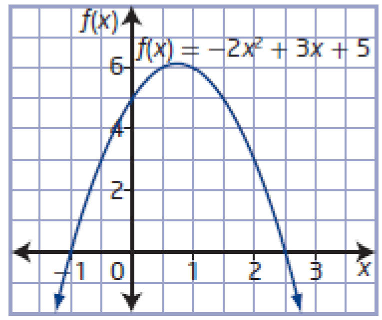
**a)** Expand $y = 7(x + 3)^2 - 41$.
$y = 7(x^2 + 6x + 9) - 41$
$y = 7x^2 + 42x + 63 - 41$
$y = 7x^2 + 42x + 22$
The function $y = 7(x + 3)^2 - 41$ is quadratic, since when expanded it is a polynomial of degree two.

**b)** Expand $y = (2x + 7)(10 - 3x)$.
$y = 20x - 6x^2 + 70 - 21x$
$y = -6x^2 - x + 70$
The function $y = (2x + 7)(10 - 3x)$ is quadratic, since when expanded it is a polynomial of degree two.

**Chapter 3 Review Page 199 Question 11**

**a)**

| $x$ | $f(x) = -2x^2 + 3x + 5$ |
|---|---|
| –1 | $f(-1) = -2(-1)^2 + 3(-1) + 5$ <br> $= 0$ |
| 0 | $f(0) = -2(0)^2 + 3(0) + 5$ <br> $= 5$ |
| 1 | $f(1) = -2(1)^2 + 3(1) + 5$ <br> $= 6$ |
| 2 | $f(2) = -2(2)^2 + 3(2) + 5$ <br> $= 3$ |
| 3 | $f(3) = -2(3)^2 + 3(3) + 5$ <br> $= -4$ |

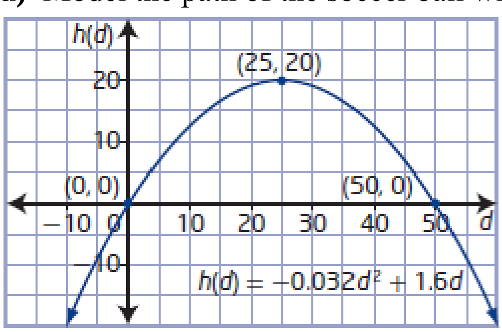



vertex: (0.75, 6.125)
axis of symmetry: $x = 0.75$
opens downward
maximum value: 6.125
domain: $\{x \mid x \in R\}$
range: $\{y \mid y \leq 6.125, y \in R\}$
$x$-intercepts: –1 and 3
$y$-intercept: 5

**b)** Answers may vary. Example: The vertex is the highest point on the parabola. The axis of symmetry is defined by the $x$-coordinate of the vertex. Since $a < 0$, the graph opens downward. The maximum value is the $y$-coordinate of the vertex. The domain is all real numbers. The range is less than or equal to the maximum value. The $x$-intercepts are where the graph crosses the $x$-axis, and the $y$-intercept is where the graph crosses the $y$-axis.

**Chapter 3 Review Page 200 Question 12**

**a)** Model the path of the soccer ball with a graph.




**b)** The maximum height of the ball is 20 m when the ball is 25 m downfield.

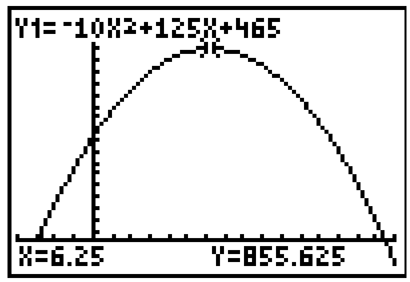
**c)** The ball hits the ground 50 m downfield.

**d)** Since both height and distance must be positive, the domain is $\{x \mid 0 \le x \le 50, x \in \mathrm{R}\}$ and the range is $\{y \mid 0 \le y \le 20, y \in \mathrm{R}\}$.

**Chapter 3 Review Page 200 Question 13**

**a)** Create a function model for the area, *A*.
$A = (31 - 2x)(5x + 15)$
$A = 155x + 465 - 10x^2 - 30x$
$A = -10x^2 + 125x + 465$

**b)** Use a graphing calculator to graph the function with window settings of *x*: [–4, 16, 1] and *y*: [–100, 1000, 50].




**c)** The portion of the graph above the *x*-axis represents the possible areas for the rectangle. So, the *x*-intercepts give the possible range of *x*-values that produce those areas.

**d)** The function has a maximum value and a minimum value (the minimum is 0 because of the context—it is an area).

**e)** The vertex gives the maximum area and the *x*-value for which it occurs.

**f)** The domain is $\{x \mid -3 \le x \le 15.5, x \in \mathrm{R}\}$ and the range is $\{A \mid 0 \le A \le 855.625, A \in \mathrm{R}\}$. The domain represents the values for *x* that will produce dimensions of a rectangle. The range represents the possible values of the area of the rectangle.

**Chapter 3 Review Page 200 Question 14**

**a)** Complete the square to write $y = x^2 - 24x + 10$ in vertex form.
$y = x^2 - 24x + 10$
$y = (x^2 - 24x) + 10$
$y = (x^2 - 24x + 144 - 144) + 10$
$y = (x^2 - 24x + 144) - 144 + 10$
$y = (x - 12)^2 - 134$
Expand $y = (x - 12)^2 - 134$ to verify the two forms are equivalent.

$y = (x - 12)^2 - 134$
$y = (x^2 - 24x + 144) - 134$
$y = x^2 - 24x + 10$

**b)** Complete the square to write $y = 5x^2 + 40x - 27$ in vertex form.
$y = 5x^2 + 40x - 27$
$y = 5(x^2 + 8x) - 27$
$y = 5(x^2 + 8x + 16 - 16) - 27$
$y = 5[(x^2 + 8x + 16) - 16] - 27$
$y = 5[(x + 4)^2 - 16] - 27$
$y = 5(x + 4)^2 - 80 - 27$
$y = 5(x + 4)^2 - 107$
Expand $y = 5(x + 4)^2 - 107$ to verify the two forms are equivalent.
$y = 5(x + 4)^2 - 107$
$y = 5(x^2 + 8x + 16) - 107$
$y = 5x^2 + 40x + 80 - 107$
$y = 5x^2 + 40x - 27$

**c)** Complete the square to write $y = -2x^2 + 8x$ in vertex form.
$y = -2x^2 + 8x$
$y = -2(x^2 - 4x)$
$y = -2(x^2 - 4x + 4 - 4)$
$y = -2[(x^2 - 4x + 4) - 4]$
$y = -2[(x - 2)^2 - 4]$
$y = -2(x - 2)^2 + 8$
Expand $y = -2(x - 2)^2 + 8$ to verify the two forms are equivalent.
$y = -2(x - 2)^2 + 8$
$y = -2(x^2 - 4x + 4) + 8$
$y = -2x^2 + 8x - 8 + 8$
$y = -2x^2 + 8x$

**d)** Complete the square to write $y = -30x^2 - 60x + 105$ in vertex form.
$y = -30x^2 - 60x + 105$
$y = -30(x^2 + 2x) + 105$
$y = -30(x^2 + 2x + 1 - 1) + 105$
$y = -30[(x^2 + 2x + 1) - 1] + 105$
$y = -30[(x + 1)^2 - 1] + 105$
$y = -30(x + 1)^2 + 30 + 105$
$y = -30(x + 1)^2 + 135$
Expand $y = -30(x + 1)^2 + 135$ to verify the two forms are equivalent.

$y = –30(x + 1)^2 + 135$
$y = –30(x^2 + 2x + 1) + 135$
$y = –30x^2 – 60x – 30 + 135$
$y = –30x^2 – 60x + 105$

**Chapter 3 Review Page 200 Question 15**

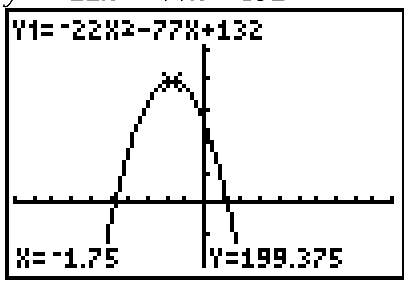
Write $f(x) = 4x^2 – 10x + 3$ in vertex form.
$f(x) = 4x^2 – 10x + 3$
$f(x) = 4(x^2 – 2.5x) + 3$
$f(x) = 4(x^2 – 2.5x + 1.5625 – 1.5625) + 3$
$f(x) = 4[(x^2 – 2.5x + 1.5625) – 1.5625] + 3$
$f(x) = 4[(x – 1.25)^2 – 1.5625] + 3$
$f(x) = 4(x – 1.25)^2 – 6.25 + 3$
$f(x) = 4(x – 1.25)^2 – 3.25$
For $f(x) = 4(x – 1.25)^2 – 3.25$, $a = 4$, $p = 1.25$, and $q = –3.25$.
vertex: (1.25, –3.25)
axis of symmetry: $x = 1.25$
minimum value: –3.25
domain: $\{x \mid x \in R\}$
range: $\{y \mid y \geq –3.25, y \in R\}$

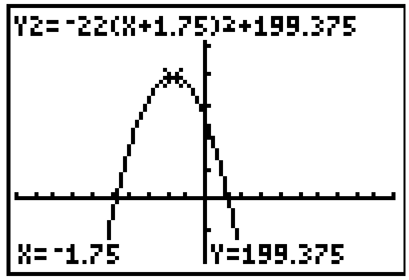
**Chapter 3 Review Page 200 Question 16**

**a)** *Amy's solution*:
$y = –22x^2 – 77x + 132$
$y = –22(x^2 – 3.5x) + 132$
$y = –22(x^2 – 3.5x – 12.25 + 12.25) + 132$
$y = –22(x^2 – 3.5x – 12.25) – 269.5 + 132$
$y = –22(x – 3.5)^2 – 137.5$
There is an error in line 2. Amy incorrectly factored –22 from 77$x$. The result should be + 3.5$x$. There is also an error in line 3. Amy should have added and subtracted the square of half the coefficient of the $x$-term, not subtracted and added the square of the coefficient of the $x$-term.
The corrected solution is shown.
$y = –22x^2 – 77x + 132$
$y = –22(x^2 + 3.5x) + 132$
$y = –22(x^2 + 3.5x + 3.0625 – 3.0625) + 132$
$y = –22(x^2 + 3.5x + 3.0625) + 67.375 + 132$
$y = –22(x + 1.75)^2 + 199.375$

**b)** Answers may vary. Example: To verify an answer, either work backward to show the functions are equivalent or use technology to show the graphs of the functions are identical.

$y = -22(x + 1.75)^2 + 199.375$
$y = -22(x^2 + 3.5x + 3.0625) + 199.375$
$y = -22x^2 - 77x - 67.375 + 199.375$
$y = -22x^2 - 77x + 132$



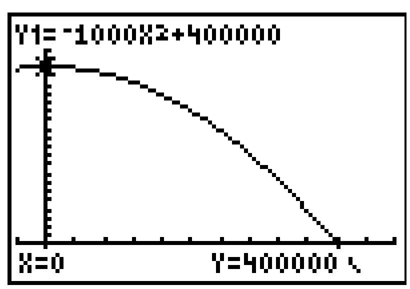




**Chapter 3 Review Page 200 Question 17**

**a)** Let $n$ represent the number of price decreases. The new price is \$40 minus the number of price decreases times \$2, or $40 - 2n$.
The new number of coats sold is 10 000 plus the number of price decreases times 500, or $10\ 000 + 500n$.
Let $R$ represent the expected revenue, in dollars.
Revenue = (price)(number of sessions)
$R = (40 - 2n)(10\ 000 + 500n)$
$R = 400\ 000 - 1000n^2$
$R = -1000n^2 + 400\ 000$

**b)** The function $R = -1000n^2 + 400\ 000$ is in vertex form.
The maximum revenue the manager can expect is \$400 000 when a coat sells for $40 - 2(0)$, or \$40.

**c)** Use a graphing calculator to graph the function with window settings of $x$: [–2, 24, 2] and $y$: [–60 000, 500 000, 20 000].




**d)** The $y$-intercept represents the maximum revenue. The positive $x$-intercept indicates the number of price decreases that will produce revenue.

**e)** For the number of price decrease, the domain is $\{x \mid 0 \le x \le 20, x \in \mathrm{R}\}$ and the range is $\{y \mid 0 \le y \le 400\ 000, y \in \mathrm{R}\}$.

**f)** Answers may vary. Example: Assume that the market research regarding price and number of coats sold holds true.

## Chapter 3 Practice Test

### Chapter 3 Practice Test Page 201 Question 1

The function $f(x) = 3(x - 9) + 6$ is linear, not quadratic. The answer is **D**.

### Chapter 3 Practice Test Page 201 Question 2

The vertex of the parabola shown is (–4, –4). Then, $p = -4$ and $q = -4$. So, the function in vertex form is $y = (x + 4)^2 - 4$. The answer is **C**.

### Chapter 3 Practice Test Page 201 Question 3

For the function $y = -6(x - 6)^2 + 6$, $a = -6$, $p = 6$, and $q = 6$. Since $a < 0$, the parabola opens downward and the range is $\{y \mid y \le 6, y \in \mathrm{R}\}$. The answer is **A**.

### Chapter 3 Practice Test Page 201 Question 4

Write $y = x^2 - 2x - 5$ in vertex form.
$y = x^2 - 2x - 5$
$y = (x^2 - 2x) - 5$
$y = (x^2 - 2x + 1 - 1) - 5$
$y = (x^2 - 2x + 1) - 1 - 5$
$y = (x - 1)^2 - 6$
The answer is **D**.

### Chapter 3 Practice Test Page 201 Question 5

The graph of $y = 1 + ax^2$ if $a < 0$ is a parabola that opens downward with vertex (0, 1). The answer is **D**.

### Chapter 3 Practice Test Page 201 Question 6

For the function $f(x) = a(x - p)^2 + q$ to have no $x$-intercepts, either $a > 0$ and $q > 0$ or $a < 0$ and $q < 0$. The answer is **A**.

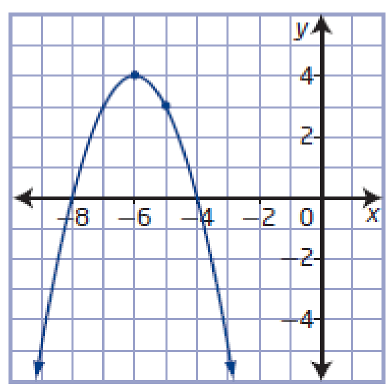
**Chapter 3 Practice Test      Page 202      Question 7**

**a)** Complete the square to write $y = x^2 – 18x – 27$ in vertex form.
$y = x^2 – 18x – 27$
$y = (x^2 – 18x) – 27$
$y = (x^2 – 18x + 81 – 81) – 27$
$y = (x^2 – 18x + 81) – 81 – 27$
$y = (x – 9)^2 – 108$

**b)** Complete the square to write $y = 3x^2 + 36x + 13$ in vertex form.
$y = 3x^2 + 36x + 13$
$y = 3(x^2 + 12x) + 13$
$y = 3(x^2 + 12x + 36 – 36) + 13$
$y = 3[(x^2 + 12x + 36) – 36] + 13$
$y = 3[(x + 6)^2 – 36] + 13$
$y = 3(x + 6)^2 – 108 + 13$
$y = 3(x + 6)^2 – 95$

**c)** Complete the square to write $y = –10x^2 – 40x$ in vertex form.
$y = –10x^2 – 40x$
$y = –10(x^2 + 4x)$
$y = –10(x^2 + 4x + 4 – 4)$
$y = –10[(x^2 + 4x + 4) – 4]$
$y = –10[(x + 2)^2 – 4]$
$y = –10(x + 2)^2 + 40$

**Chapter 3 Practice Test      Page 202      Question 8**




**a)** The coordinates of the vertex are (–6, 4).
The equation of the axis of symmetry is $x = –6$.
The graph has a maximum value of 4, since the parabola opens downward. The domain is $\{x \mid x \in R\}$ and the range is $\{y \mid y \le 4, y \in R\}$.
The $x$-intercepts are –8 and –4.

**b)** Since the vertex is located at (–6, 4), $p = –6$ and $q = 4$.
So, the function is of the form $y = a(x + 6)^2 + 4$.
Substitute (–5, 3) and solve for $a$.
$3 = a(–5 + 6)^2 + 4$
$3 = a + 4$
$a = –1$
The quadratic function in vertex form is $y = –(x + 6)^2 + 4$.

## Chapter 3 Practice Test Page 202 Question 9

**a) i)** To graph $f(x) = 5x^2$, transform the graph of $f(x) = x^2$ by multiplying the $y$-values by a factor of 5.

**ii)** To graph $f(x) = x^2 – 20$, transform the graph of $f(x) = x^2$ by translating 20 units down.

**iii)** To graph $f(x) = (x + 11)^2$, transform the graph of $f(x) = x^2$ by translating 11 units to the left.

**iv)** To graph $f(x) = -\frac{1}{7}x^2$, transform the graph of $f(x) = x^2$ by multiplying the $y$-values by a factor of $\frac{1}{7}$ and reflecting in the $x$-axis.

**b)** Answers may vary. Examples:

**i)** Compared to the graph of $f(x) = x^2$, the vertex of the functions in part a) ii) and iii) will be different because these graphs are translated vertically or horizontally, respectively.

**ii)** Compared to the graph of $f(x) = x^2$, the axis of symmetry of the functions in part a) ii) and iii) will be different because these graphs are translated vertically or horizontally, respectively.

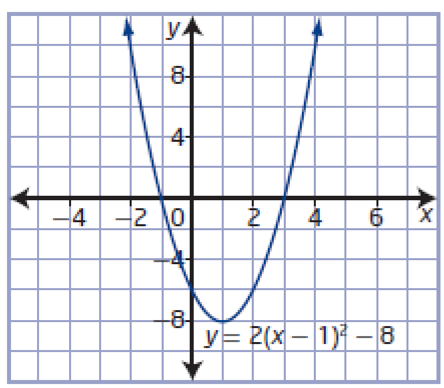
**iii)** Compared to the graph of $f(x) = x^2$, the range of the functions in part a) ii) and iv) will be different because these graphs are translated vertically or reflected in the $x$-axis, respectively.

## Chapter 3 Practice Test Page 202 Question 10

For $f(x) = 2(x – 1)^2 – 8$, $a = 2$, $p = 1$, and $q = –8$.

To sketch the graph of $f(x) = 2(x – 1)^2 – 8$, transform the graph of $f(x) = x^2$ by

- multiplying the $y$-values by a factor of 2
- translating 1 unit to the right and 8 units down



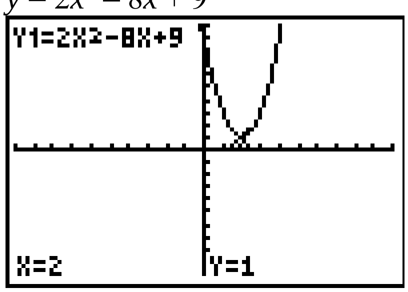

| | |
|---|---|
| Vertex | (1, –8) |
| Axis of Symmetry | $x = 1$ |
| Direction of Opening | upward |
| Domain | $\{x \mid x \in R\}$ |
| Range | $\{y \mid y \geq -8, y \in R\}$ |
| $x$-Intercepts | –1 and 3 |
| $y$-Intercept | –6 |

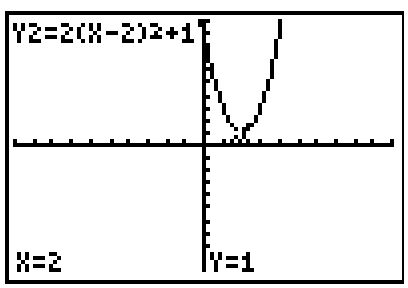
**Chapter 3 Practice Test Page 202 Question 11**

**a)** Given steps of completing the square:
$y = 2x^2 – 8x + 9$
$y = 2(x^2 – 8x) + 9$
$y = 2(x^2 – 8x – 64 + 64) + 9$
There is an error in line 2. The leading coefficient of 2 was not factored out of $8x$.
There is an error in line 3. The square of half the coefficient of the $x$-term should have added and subtracted.
The corrected lines are shown.
$y = 2x^2 – 8x + 9$
$y = 2(x^2 – 4x) + 9$
$y = 2(x^2 – 4x + 4 – 4) + 9$

**b)** Completing the process from part a):
$y = 2x^2 – 8x + 9$
$y = 2(x^2 – 4x) + 9$
$y = 2(x^2 – 4x + 4 – 4) + 9$
$y = 2[(x^2 – 4x + 4) – 4] + 9$
$y = 2[(x – 2)^2 – 4] + 9$
$y = 2(x – 2)^2 – 8 + 9$
$y = 2(x – 2)^2 + 1$

**c)** To verify an answer, either work backward to show the functions are equivalent or use technology to show the graphs of the functions are identical.
$y = 2(x – 2)^2 + 1$
$y = 2(x^2 – 4x + 4) + 1$
$y = 2x^2 – 8x + 8 + 1$
$y = 2x^2 – 8x + 9$



**Chapter 3 Practice Test Page 202 Question 12**

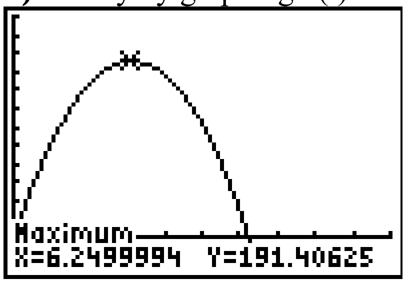
**a)** Write $C(v) = 0.004v^2 – 0.62v + 30$ in vertex form.
$C(v) = 0.004v^2 – 0.62v + 30$
$C(v) = 0.004(v^2 – 155v) + 30$
$C(v) = 0.004(v^2 – 155v + 6006.25 – 6006.25) + 30$
$C(v) = 0.004[(v^2 – 155v + 6006.25) – 6006.25] + 30$
$C(v) = 0.004[(v – 77.5)^2 – 6006.25] + 30$
$C(v) = 0.004(v – 77.5)^2 – 24.025 + 30$
$C(v) = 0.004(v – 77.5)^2 + 5.975$
In this form, the vertex is read easily as (77.5, 5.975).
The most efficient speed at which the car should be driven is 77.5 km/h.

**b)** Answers may vary. Example:
Without graphing a quadratic function in standard form you can determine the vertex, axis of symmetry, maximum or minimum value, domain, range, and $y$-intercept. Find the $x$-coordinate of the vertex and the axis of symmetry with $x = \frac{-b}{2a}$. Use that value to find the $y$-coordinate of the vertex and the maximum or minimum value. The domain is restricted by the situation. The range is restricted by the situation and the maximum or minimum value. The $y$-intercept is the value of $c$.
Without graphing a quadratic function in vertex form you can determine the vertex, axis of symmetry, maximum or minimum value, domain, range, and $y$-intercept. The coordinates of the vertex are given as $(p, q)$. The axis of symmetry is $x = p$. The maximum or minimum value is $q$. The domain is restricted by the situation. The range is restricted by the situation and the maximum or minimum value. The $y$-intercept is the value of the function for $x = 0$.

**Chapter 3 Practice Test Page 203 Question 13**

Answers may vary. Examples:
**a)** Complete the square to find the maximum height of the flare and when it occurred.
$h(t) = –4.9t^2 + 61.25t$
$h(t) = –4.9(t^2 – 12.5t)$
$h(t) = –4.9(t^2 – 12.5t + 39.0625 – 39.0625)$
$h(t) = –4.9[(t^2 – 12.5t + 39.0625) – 39.0625]$
$h(t) = –4.9[(t – 6.25)^2 – 39.0625]$
$h(t) = –4.9(t – 6.25)^2 + 191.40625$
The maximum height of the flare is 191.40625 m and this occurs 6.25 s after the it is fired.

**b)** Verify by graphing $h(t) = -4.9t^2 + 61.25t$ and finding the vertex.




Alternatively, use $t = \frac{-b}{2a}$ to find the $t$-coordinate of the vertex.

$t = \frac{-61.25}{2(-4.9)}$

$t = 6.25$

Substitute $t = 6.25$ into $h(t) = -4.9t^2 + 61.25t$ to find the $h$-coordinate of the vertex.

$h(6.25) = -4.9(6.25)^2 + 61.25(6.25)$

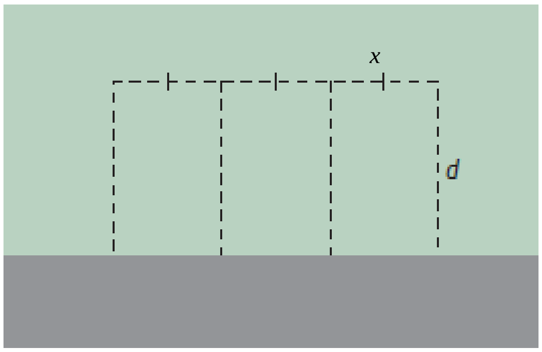
$h(6.25) = 191.40625$

The vertex is located at (6.25, 191.40625).

**Chapter 3 Practice Test Page 203 Question 14**

**a)** Let $x$ represent the width of the small rectangle.

Write an expression for the total amount of fencing: $24 = 3x + 4d$, or $x = 8 - \frac{4}{3}d$.




Write a function for the total area.

$A(d) = d(3x)$

$A(d) = d\left[3\left(8 - \frac{4}{3}d\right)\right]$

$A(d) = d(24 - 4d)$

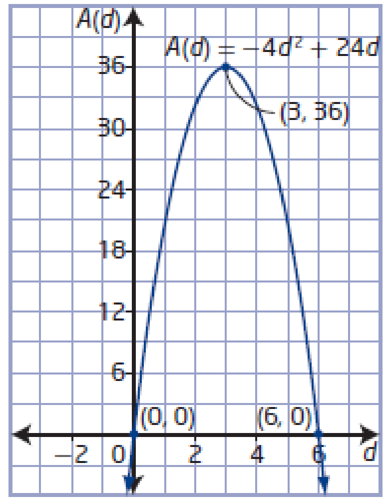
$A(d) = 24d - 4d^2$

$A(d) = -4d^2 + 24d$

**b)** The function $A(d) = -4d^2 + 24d$ is quadratic, since it is a polynomial of degree two.

**c)** Answers may vary. Example: Graph the function by first completing the square. Then, determine the coordinates of the *y*-intercept and apply symmetry.

$A(d) = -4d^2 + 24d$
$A(d) = -4(d^2 - 6d)$
$A(d) = -4(d^2 - 6d + 9 - 9)$
$A(d) = -4[(d^2 - 6d + 9) - 9]$
$A(d) = -4[(d - 3)^2 - 9]$
$A(d) = -4(d - 3)^2 + 36$
Let $x = 0$.
$A(0) = -4(0 - 3)^2 + 36$
$A(0) = 0$
The point is (0, 0), and its corresponding point is (6, 0).




**d)** The coordinates of the vertex are (3, 36). This represents a maximum of area of 36 m$^2$ when the distance from the wall is 3 m.

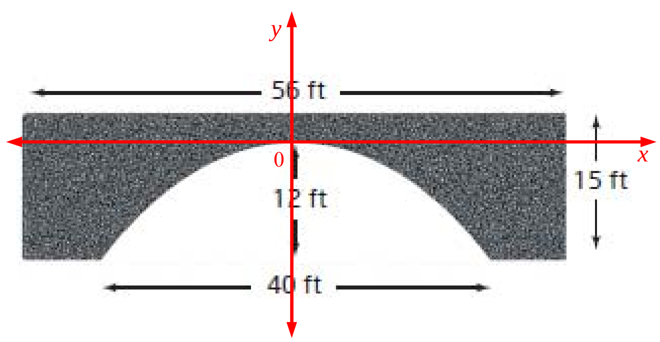
**e)** Since neither the distance from the wall or area can be negative, the domain is $\{d \mid 0 \le d \le 6, d \in \mathrm{R}\}$ and the range is $\{A \mid 0 \le A \le 36, A \in \mathrm{R}\}$.

**f)** The function has a maximum value of 36 when $d = 3$, and the function has a minimum value of 0 when $d = 0$ or $d = 6$.

**g)** Answers may vary. Example: Assume that all the fencing is used.

**Chapter 3 Practice Test Page 203 Question 15**

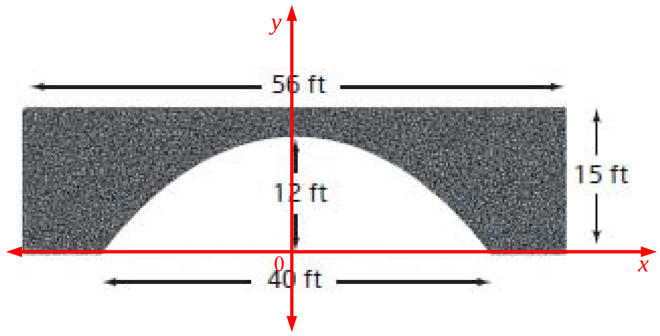
**a)** The location of the origin is the top of the opening under the bridge. Let *x* and *y* represent the horizontal and vertical distances from the high point of the opening, respectively. Then, the vertex is at (0, 0) and the quadratic function is of the form $y = ax^2$. From the diagram, another point on the parabola is (20, –12). Use these coordinates to find *a*.




$-12 = a(20)^2$
$-12 = 400a$
$a = -\dfrac{3}{100}$

A quadratic function that represents the shape of the arch is $y = -\dfrac{3}{100}x^2$.

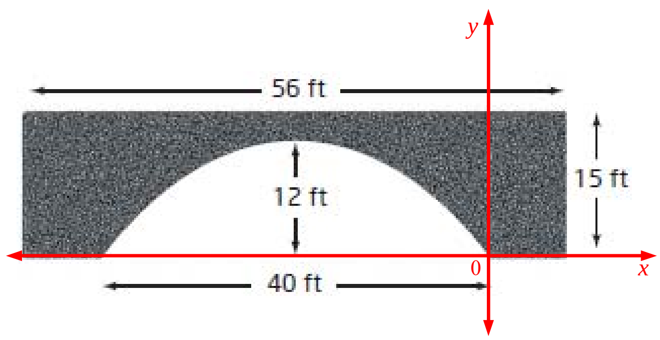
**b)** The location of the origin is on the ground at the midpoint of the opening. Let $x$ and $y$ represent the horizontal and vertical distances, respectively. Then, the vertex is at (0, 12) and the quadratic function is of the form $y = ax^2 + 12$. From the diagram, another point on the parabola is (20, 0).
Use these coordinates to find $a$.




$$0 = a(20)^2 + 12$$
$$-12 = 400a$$
$$a = -\frac{3}{100}$$

A quadratic function that represents the shape of the arch is $y = -\frac{3}{100}x^2 + 12$.

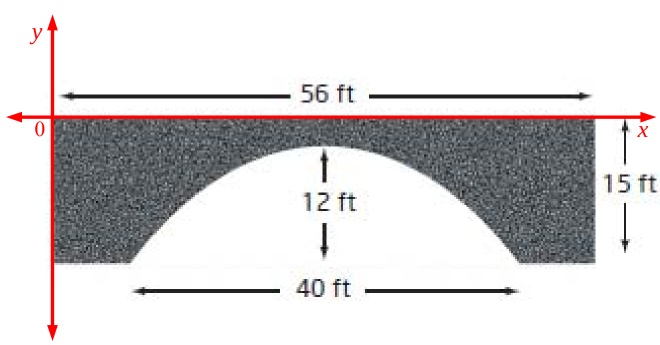
**c)** The location of the origin is at the base of the bridge at the right side of the opening. Let $x$ and $y$ represent the horizontal and vertical distances, respectively. Then, the vertex is at (–20, 12) and the quadratic function is of the form $y = a(x + 20)^2 + 12$.
From the diagram, another point on the parabola is (0, 0). Use these coordinates to find $a$.




$$0 = a(0 + 20)^2 + 12$$
$$-12 = 400a$$
$$a = -\frac{3}{100}$$

A quadratic function that represents the shape of the arch is $y = -\frac{3}{100}(x + 20)^2 + 12$.

**d)** The location of the origin is on the left side at the top surface of the bridge. Let $x$ and $y$ represent the horizontal and vertical distances, respectively. Then, the vertex is at (28, –3) and the quadratic function is of the form $y = a(x - 28)^2 - 3$.
From the diagram, another point on the parabola is (8, –15). Use these coordinates to find $a$.




$$-15 = a(8 - 28)^2 - 3$$
$$-12 = 400a$$
$$a = -\frac{3}{100}$$

A quadratic function that represents the shape of the arch is $y = -\frac{3}{100}(x - 28)^2 - 3$.

**Chapter 3 Practice Test Page 203 Question 16**

**a)** Let $n$ represent the number of price decreases. The new price is \$2.25 minus the number of price decreases times \$0.05, or $2.25 – 0.05n$.
The new number of energy bars sold monthly is 120 plus the number of price decreases times 8, or $120 + 8n$.
Let $R$ represent the expected revenue, in dollars.
Revenue = (price)(number of sessions)
$R = (2.25 – 0.05n)(120 + 8n)$
$R = 270 + 18n – 6n – 0.4n^2$
$R = –0.4n^2 + 12n + 270$

**b)** Complete the square to find the vertex.
$R = –0.4n^2 + 12n + 270$
$R = –0.4(n^2 – 30n) + 270$
$R = –0.4(n^2 – 30n + 225 – 225) + 270$
$R = –0.4[(n^2 – 30n + 225) – 225] + 270$
$R = –0.4[(n – 15)^2 – 225] + 270$
$R = –0.4(n – 15)^2 + 90 + 270$
$R = –0.4(n – 15)^2 + 360$
The maximum revenue the manager can expect is \$360 per month when an energy bar sells for $2.25 – 0.05(15)$, or \$1.50.

**c)** Answers may vary. Example: Assume that the information regarding price and number of energy bars sold holds true.